Abstract

This paper deals with the methods of physical modelling of Flight Control Systems (FCS) by means of Dynamically Similar free-flying Models (DSM) for the investigation of stability and controllability of aircraft at subsonic flight speeds. The subsonic flight regime allows us to avoid Mach number similarity considerations. The large scale of the DSM meets autosimilarity of Reynolds numbers, whilst Froude similarity is assured during the development and manufacturing of the DSM. This paper proves the presence of necessary and sufficient conditions of similarity for the FCS of an aircraft, and that of its DSM. The existence of necessary conditions have been proved both mathematically (by means of the \( \pi \) Theorem from the theory of dimensions), and with equations involving physical quantities. The generalised scale coefficients for transitioning from the FCS’s aircraft gain factors, to those of the FCS of the DSM have been obtained, and it is shown that the coefficients depend only on the linear scale of the DSM.

NOMENCLATURE

- \( c \): mean aerodynamic chord
- \( C_1 \ldots C_5 \): coefficient of linearised equations
- \( C_D \): drag coefficient
- \( C_L \): aeroplane lift coefficient
- \( C_M \): pitching moment
- \( \partial C_M / \partial \delta \)
- \( \partial C_M / \partial \alpha \)
- \( C_{M0} \)
- \( g \): acceleration due to gravity
- \( G_\theta \): gain of FCS by pitch angle
- \( G_\theta \): gain of FCS by integral of pitch angle
- \( I_Y \): moment of inertia about the \( Y \) body axis
- \( K_i \): scale coefficient (\( i = C_1 \ldots C_5, L, g, \theta, \alpha, \rho \))
- \( L \): length, m
- \( m \): mass
- \( q \): pitch rate, deg/s
- \( q_c \): dimensionless pitch rate \( q / 2V \)
- \( S \): wing surface, m²
- \( t \): time, s
- \( V \): velocity, ms⁻¹

Greek letters

- \( \alpha \): angle-of-attack, deg
- \( \delta \): surface deflection, deg
- \( \pi_0 \): nondimension coefficients of similarity
- \( \theta \): pitch angle, deg
- \( \rho \): mass density, kgm⁻³
- \( \xi \): damping ratio

\[ \text{[ ]} \] designates the dimensions of a quantity
Subscripts
H altitude
M model
N nature

Superscripts
• derivative with respect to time

1.0 INTRODUCTION

There are several methods known for modelling the Flight Control System (FCS) of a new aircraft: mathematical modelling, partial system test and by means of a flying simulator.

Mathematical simulation is usually applied at an early stage of FCS design. It has considerable advantages, such as: the results of a simulation are obtained in a very short time period, the possibility to investigate large number of variants, and its low cost. Nevertheless, together with these advantages, mathematical simulation has also drawbacks: the preparation of the mathematical model for such a complex object as an aircraft is a quite difficult problem.

Computational methods in aerodynamics may produce large errors during the calculation of an aircraft’s performance, whilst wind tunnel tests can not reproduce many of the complex manoeuvres of an aircraft; moreover, the physical boundaries of the walls that surround the model, the presence of a suspension and attachments, all produce additional errors in the flight parameters investigated.

Assumptions and limitations introduced in the preparation of the mathematical model do not allow for an adequate representation of the real system ‘Aircraft-FCS’. In particular, discrepancies arise during the development of tests for an aircraft of a new generation or when exploring new flight regimes. In these cases new physical phenomena may be involved for which a mathematical description is not yet available; for example, the aircraft’s behaviour at angles of incidence beyond stall, which is very difficult to describe.

A partial system test uses real FCS hardware together with the mathematical model of the controlled object. This method, excludes errors derived from the various assumptions in deriving the mathematical model of the FCS. In this case, the model of the aircraft remains without change and it contains all the errors introduced during the development of the mathematical modelling method.

The DSM test is a physical simulation. This simulation is different from the other kinds of simulation (mathematical, partial system etc.). The model has the same physical nature as a full-scale object and it is the cardinal distinction. The method of physical simulation allows to establish a quantitative ratio among parameters of a full-scale object without the solution of differential equations which ones describe a process (flight).

Flight simulation by means of a flying simulator is widely used for the investigation of new aircraft. For example, the flying simulator NT-33A used for the F-15, YF-16, YF-17, YF-18 aeroplanes[1]. Shortcomings in the flight performance of these aeroplanes were detected at an early stage of flight-testing. The general principles of design of a flying simulator are the following: the basic aircraft configuration is selected; additional modules are introduced in the control system for examining the modelling characteristics of the FCS. These units have software, which may change the control laws and dynamics of the basic aircraft. This flying simulator is very good for crew training and for the estimation by test pilots of the flight performance of a new aircraft. However, as for the addition of FCS modules that have a mathematical model of the investigated system, one must be aware that all errors of mathematical model also appear in the flying simulator.

In scientific and research centres of England and USA, DSM are used since 1940. In 1940 in England, the tests of freely flying model for research of the aerodynamic characteristics were conducted. In 1941-1946 in USA researches on models of seaplanes was held[2]. The Lockheed Company has received a significant part of the flight characteristics of an aeroplane XF by means of models started in free flight on a high altitude. The part of the flight characteristics of aeroplanes XF-91, XF-92A, XF4D-1 and Bell X-2 was obtained by means of DSM[3]. The models were used for development of aeroplanes F-104 and F-15. In 2000, the tests of model X-36 were held in USA.[4] In USSR and later in Russia DSM were applied for research of newest aeroplanes. So, on the exhibition MAKS-92 and MAKS-93 the models of aeroplanes MiG-29, Su-27 and Su-35 making about 50 exploratory flights everyone were shown.

Any simulation (similarity of a model and an aeroplane) is approximated, so the presentation of investigated phenomenon in its entirety is possible only for full-sized aeroplane flight conditions. On the other hand, the fully exact presentation of all aeroplane parameters on its model does not always provide more exact data. It is necessary to have precise understanding about allowable differences of the model and the aeroplane for the suitable experiment realisation.

For example, two cases can be investigated. A subject of investigation is the aeroplane, which is flying on a certain regime (the given altitude and speed of flight). The questions are:

1. If the aeroplane flying on the determined regime (altitude and speed) is a subject of research, is it possible to consider as its model the same aeroplane but which is flying on the other regime (at other altitude, or at the same altitude, but with other speed)?
2. Can the aeroplane of other type (other weight and the form), flying on the same or other regime, be considered as a model?

The unequivocal answer for these two questions does not exist, because they do not include enough information for decision-making. It is impossible to speak about similitude of two objects (two aeroplanes or two regimes of their flight) without defining the type of similitude of the phenomena that is being investigated. Model and conditions, in which it is situated, can reproduce some authentically properties of the full-scale object and at the same time, it does not correspond to a nature of a lot of other characteristics.

In the first case, for example, if radar reflection power of an aeroplane is investigated, practically, it has no value, considering the aeroplane flies at speed of 150 or 200ms⁻¹. If the stall or spin process are investigated, the flight speed has basic and defining value. With a flight speed about 150ms⁻¹, the aeroplane can have enough stability and controllability, but to fly with a speed about 50ms⁻¹ is unable. With a flight speed of about 150ms⁻¹ near to the Earth surface, the aeroplane is stable and controlled, but the same speed for a flight altitude above 10,000m, is close to critical. In this case, despite of apparent full conformity of a nature and model, it is observed that flight of the aeroplane with another speed, as its own model is inadmissible for researching of such phenomena.

In the second case, accurate definition of a modelling problem is required too. For example, for researching of a man-autopilot system and also for training of pilots of heavy aeroplanes, more light aeroplanes are used as simulators. But, into their flight control system, the special units, which change dynamic characteristics of aeroplane-model and make it similar to a heavy aeroplane, are added. In this case, the full-scale object is simulated by model, which does not have the geometrical similitude with it, but precisely enough and authentically represents its behaviour.

So, to assert that one object is or not model of other object and to determine conditions of similitude, it is possible to precise only with reference to certain class of modelled phenomena. DSM should correspond to a full-sized aeroplane by the parameters, which provide the essential effect on researched process. In this case, the necessity to get the maximum conformity of the model to the aeroplane by the other parameters (for example, to be aimed to make its sizes as much as possible) is absent. On the contrary, the capability of the optional selection of some scales and parameters of the model, which are not essential for the researched process, considerably facilitates the model design and the flight test program development.

Dynamically similar free-flying models (DSM) may be utilised in
practice in each step of the process of generating new aircraft, when data and performances are entirely different from a previous generation.

A DSM is an autonomous, pilotless, research and development multiple mission aircraft. It can carry out automatic or remotely controlled flights, according to a present program, and has the capability to record all flight relevant information during a test. The DSM is geometrically similar to the aircraft under investigation. Its mass, moment of inertia and other parameters ensure exact performance similarity, including the case when it is necessary to ensure an adequate simulation of aeroelastic processes. DSM is a physical model of an aircraft, and it excludes all errors that the non-linear or linearised mathematical model of that aircraft may have. It is of most importance in the investigation of new flight regimes: for example, manoeuvre such as the well know ‘Cobra’ (reaching an angle of attack of more than 90° without stalling). The development of a mathematical model for predicting behaviour of aircraft is as mentioned above a very difficult problem; however, a DSM may solve this problem readily.

Regimes which attain an angle of attack equal to 90° are pre-set at the FCS of the DSM, and after flight tests, better control laws may be selected. The DSM is launched from a carrier (aircraft or helicopter) or from ground ramps with the help of a solid-propellant ascent stage. After stage separation, the DSM accomplishes pre-programmed manoeuvres; in case of an emergency, or after flight program completion, a parachute and soft landing system performs a proper recovery. It may also recover the DSM from critical or uncontrolled flight regimes, can stabilise and decelerate its motion and at the landing moment, the same system will decrease vertical speed for touchdown and counteracts ground wind effects. After processing the flight’s information, performing turn-round servicing and on-board systems readjusting, in accordance with a new flight program, further flight experiments may be conducted. Thus, dictated by issues of test pilot safety as well as by economic considerations, there is a clear need for the most realistic simulation of the dynamics of flight, specially in the exploration of critical flight regimes for new aircraft, by means of such type of dynamically similar free-flying models.

At an early stage in the development of studies applying free-flying models, investigators were only interested in the aerodynamic characteristics of the aircraft, without taking much into account the influence of the FCS on these characteristics, and as is well known modellings of FCS through DSM have not been utilised. The more frequent development and the growth of research efforts on active control systems has provided a need to further simulate FCS, specially since a similarity transfer function between the FCS of the real aircraft, with that of its DSM, is either incomplete or simply absent.

### 2.0 SIMILARITY CONDITIONS OF THE FCS OF AN AIRCRAFT AND ITS DSM

Physical phenomena, processes or systems are similar if in the analogous time moments and in the analogous space points, the variable value which characterise the state of one system, are proportional to the corresponding variable values of other system. The proportionality factors for each of these values are named as scales of similarity.

The mandatory condition of a simulation realisation on DSM is the providing of identical flow around the surfaces of the model and the aeroplane. It means a providing of a more exact correspondence of the forces acting on the aeroplane and on the model, i.e. the dynamic similarity.

To establish similarity conditions between the FCS of an aircraft and that of its DSM, it is necessary to confront two phenomen: first, the behaviour of the FCS of the real aircraft, and second, the behaviour of the FCS of the free flying model. Furthermore, it is necessary to examine how it is possible to corroborate similarity conditions for both systems.

Peter Smith (Aerodynamic Theory, Volume 1, p 27) states that: “π – theorem plays so important a part in many problem involving dimensions and kinematics similitude. It is the equivalent in mathematical language of the statement that any algebraic form expressing a relation between and among a related series of physical quantities may be adequately and fully represented as a series of terms involving π – functions.” Where π – functions are the criteria of a similarity. In other words, the π – theorem may be formulated in the following way: The functional relation among the process parameters can be presented as a relation among the criteria, taken from them.

Necessary and sufficient conditions must be met for the comparison of such phenomenon to be valid. Necessary conditions include: the presence of a similarity criterion which must be numerically equivalent for similar phenomenon, sufficient accuracy in the geometrical similarity of the aircraft and its model, and finally, similarity of the initial and boundary conditions of the processes to be compared.

### 3.0 NECESSARY CONDITIONS

The purpose of this paragraph is the obtaining of similarity criteria, which must be numerically equivalent for similar phenomena. These criteria should be obtained by means of stringent mathematical transformations of differential equations which describe the ‘Aeroplane – FCS’ system.

The method presented in this section is valid for nth order and non-linear equations. The simplicity of the example does not influence the results, but allows the obtaining of it in a faster and more obvious way. Besides, it is possible to use linearised equations because it is not fundamental for getting the technique of the criteria obtaining. For a clear demonstration of necessary conditions for similarity between of the FCS of an aircraft and that of its DSM it is possible to examine the simple longitudinal, short-period motion of the system ‘Aircraft-FCS’, described by the following equations:

\[ (s^2 + C_1 s) \ddot{e} + C_2 s \dot{e} + C_3 \dot{e} + C_4 e + C_5 \dddot{e} + C_6 \dddot{e} = 0 \]

\[ -s \dot{e} + (s + C_7) \dddot{e} = 0 \]

\[ \dddot{e} = G_9 \dot{e} + G_{10} e + \frac{1}{s} G_{11} \]

(1)
Let us write the system of Equations (1) in the form:

\[ \varphi_1 + \varphi_2 + \varphi_3 + \ldots + \varphi_{12} = 0 \]

Where:

\[
\begin{align*}
\varphi_1 &= s^4 \dot{\varepsilon}; \quad \varphi_2 = s^2 C_4 \dot{\varepsilon}; \\
\varphi_3 &= s^2 C_1 \dot{\varepsilon}; \quad \varphi_4 = s^2 C_1 C_4 \dot{\varepsilon}; \\
\varphi_5 &= s^2 \dot{\varepsilon}; \quad \varphi_6 = s^2 C_2 \dot{\varepsilon}; \\
\varphi_7 &= s^2 C_1 G_4 \dot{\varepsilon}; \quad \varphi_8 = s^2 C_1 C_4 G_4 \dot{\varepsilon}; \\
\varphi_9 &= s^2 C_1 \dot{G}_4 \dot{\varepsilon}; \quad \varphi_{10} = s C_1 C_4 \dot{G}_4 \dot{\varepsilon}; \\
\varphi_{11} &= s C_1 \dot{G}_4 \dot{\varepsilon}; \quad \varphi_{12} = C_1 C_4 G_4 \dot{\varepsilon};
\end{align*}
\]

The symbols of differentiation and integration have not dimension and do not influence conditions of proportionality. Therefore, the members of the equations \( \varphi_\text{L} \), which contain these symbols, it is possible to replace with their analogues \( \varphi_\text{L} \), which are named as integral-analogue(13), i.e.:

\[
\frac{d^nx}{dy^y} \rightarrow \frac{x}{y} \quad \text{and} \quad \int x \, dy \rightarrow \frac{x \, y}{y}
\]

The rules of definition of the similarity criteria of processes, which are described by equations with an inhomogeneous function, exist(13). For example, if \( \varphi = U \cdot \sin \omega \cdot t \), then \( \pi = U \) and the additional criterion of a similarity \( \pi_{\text{M}} = \omega \cdot \tau \) is entered.

In order to comply with the similarity criterion by the integral-analogue method is possible to write:

\[
\varphi_1' = \frac{\dot{\varepsilon}}{t}; \quad \varphi_2' = C_1 \frac{\dot{\varepsilon}}{t}; \quad \varphi_3' = C_1 \frac{\dot{\varepsilon}}{t};
\]

\[
\varphi_4' = C_1 C_4 \frac{\dot{\varepsilon}}{t}; \quad \varphi_5' = C_1 \frac{\dot{\varepsilon}}{t} \ldots
\]

\[
\varphi_6' = C_1 G_4 \frac{\dot{\varepsilon}}{t}; \quad \varphi_7' = C_1 \frac{\dot{\varepsilon}}{t};
\]

\[
\varphi_8' = C_1 C_4 G_4 \frac{\dot{\varepsilon}}{t}; \quad \varphi_9' = C_1 \frac{\dot{\varepsilon}}{t};
\]

\[
\varphi_{10}' = C_1 C_4 \frac{\dot{\varepsilon}}{t}; \quad \varphi_{11}' = C_1 \frac{\dot{\varepsilon}}{t};
\]

\[
\varphi_{12}' = C_1 C_4 G_4 \frac{\dot{\varepsilon}}{t}
\]

and

\[
\begin{align*}
\delta_1 &= \frac{\varphi_1'}{\varphi_1} = C_4 t; \\
\delta_2 &= \frac{\varphi_2'}{\varphi_1} = C_1 t; \\
\delta_3 &= \frac{\varphi_3'}{\varphi_1} = C_1 C_4 t^2; \ldots \\
\delta_4 &= \frac{\varphi_4'}{\varphi_1} = C_1 \frac{\dot{G}_4}{t}; \ldots \\
\delta_5 &= \frac{\varphi_5'}{\varphi_1} = C_1 \frac{\dot{G}_4}{t}; \\
\delta_6 &= \frac{\varphi_6'}{\varphi_1} = C_1 C_4 G_4 \frac{\dot{\varepsilon}}{t}; \ldots
\end{align*}
\]

As far as the similarity criterion these are non-dimensional quantities, therefore the product of the scale coefficients of this criterion must be equal unity;

\[
\begin{align*}
K_{C_4} \cdot K_1 &= 1; \quad K_{C_1} \cdot K_1 &= 1; \\
K_{C_4} \cdot K_{C_1} &= 1; \quad K_{C_4} \cdot K_{C_4} = 1; \\
K_{C_4} \cdot K_{C_4} &= 1; \quad K_{C_1} \cdot K_{C_4} = 1; \\
K_{C_4} \cdot K_{C_4} &= 1; \quad K_{C_1} \cdot K_{C_4} = 1; \\
K_{C_4} \cdot K_{C_4} &= 1; \quad K_{C_4} \cdot K_{C_4} = 1; \\
K_{C_4} \cdot K_{C_4} &= 1; \quad K_{C_4} \cdot K_{C_4} = 1;
\end{align*}
\]

It is known that it is impossible to provide for full aerodynamic similarity in a single model test’s flight, in other words, simultaneous equality of Reynolds, Froude and Mach numbers for the real aircraft and its DSM is not within reach(10, 14).

The Froude, Reynolds and Mach criteria impose the conflicting restriction on the scales of linear sizes \( K_L \) and speeds \( K_V \). Let us consider, for example, the obtaining of the speed scale from the Froude criterion:

\[
\frac{V^2_L}{g N L_N} = \frac{V^2_F}{g M L_M} = F_r; \quad \text{Fr} = \text{idem}
\]

where ‘idem’ is the latin word and means the same.

The airplane and the model are in the Earth’s gravitational field, therefore \( g_N = g_M \) and

\[
\frac{V^2_L}{V^2_F} = \frac{K_L}{K_F}; \quad \text{i.e.} \quad K_V = \sqrt{\frac{K_L}{K_F}} \ldots (8)
\]

Analogously, from Reynolds criterion;

\[
\frac{V_N}{V_M} = \frac{M}{N}
\]

where \( V \) is the coefficient of kinematics viscosity, it is possible to receive;

\[
K_V = \frac{a_N}{a_M}
\]

Moreover, the ratio;

\[
\frac{V_M}{V_N}
\]

varies in a narrow range (according to the table of standard atmosphere the \( V \) is equal \( v = 1.46 \times 10^{-3} \text{ m}^2 \text{ s}^{-3} \) near the ground, and \( v = 3.9 \times 10^{-3} \text{ m}^2 \text{ s}^{-3} \) at 11 kilometres altitude).

For realisation of the similarity by the Mach criterion;

\[
\frac{V_N}{V_M} = \frac{V_M}{V_N}
\]

it is necessary to provide the speed scale,

\[
K_V = \frac{a_M}{a_N}
\]

The sound speed \( a \) varies as altitude function in the smaller \( v \) range. The ratio of sound speed \( a \) on difference altitude is equal.

\[
\frac{a_M}{a_N} = 1
\]

Three ratios between the speed similarity scales and the linear sizes;

\[
\begin{align*}
K_{V,Fr=idem} &= \sqrt{K_L}; \quad K_{V,Re=idem} = \frac{V_M}{V_N} \frac{1}{K_L}; \ldots (9)
\end{align*}
\]

can be executed simultaneously only in a trivial case,

\[
K_V = K_L = 1
\]

i.e. at experiments on the aeroplane or on the model with the same aeroplane sizes.

However, it is possible to neglect similarity of Mach numbers for the simulation of stability and controllability, when the airspeed is less than the velocity of sound. In the case of a large-scale DSM model, it is possible to sustain the self-similarity of Reynolds number. Therefore, for the exploration of stability and controllability behaviour it is necessary observe first of all the Froude criterion.

In aerodynamic experiments, the Newton criterion;

\[
\frac{F}{\pi L} = Ne
\]
is always used. It is interpreted as a proportionality support of the acting aerodynamic and inertial forces of body. From this criterion, it is possible to receive:

\[ K_m = K_l^3 \]

by means of the above-stated transformations.

The equation expressing the relationship between inertial and weight forces (\( Fr = \text{idem} \)), follows:

\[ \begin{align*}
L_m &= L_N \cdot K_l^2, \\
S_m &= S_N \cdot K_l, \\
t_m &= t_N \cdot \sqrt{K_l}, \\
V_m &= V_N \cdot \sqrt{K_l}, \\
m_m &= m_N \cdot K_l^3.
\end{align*} \]

Acceleration density \( \rho_m = \rho_N \cdot K_l \).

Angular velocity \( \omega_m = \omega_N / \sqrt{K_l} \).

Moment of inertia \( I_m = I_N \cdot K_l^3 \).

In practice, after the linear scale is determined, the model mass is being obtained by an equation;

\[ m_m = m_N \cdot K_l^3 \]

The model mass is adjusted by means of the selection of equipment weight and, if it is necessary, by means of additional masses. The fulfilment of the inertial scale;

\[ I_m = I_N \cdot K_l^3 \]

is obtained by appropriate allocation of the equipment and by using additional masses in the model construction.

With Equation (7) and;

\[ K_l = \sqrt{K_l} \]

from Equation (10) it is possible to calculate the scale coefficients for Equation (1)

\[ \begin{align*}
K_{c_1} &= K_{c_4} = K_{c_5} = 1/ \sqrt{K_l}, \\
K_{c_2} &= 1/ K_l, \\
K_{g_e} &= 1, \\
K_{g_{ph}} &= \sqrt{K_l}, \\
K_{g_{ph}} &= 1/ \sqrt{K_l}.
\end{align*} \]

The foregoing method of obtaining the scale coefficients is a formal mathematical method; therefore, it is necessary to verify by means of other procedures, which do take into account the physical nature of the occurring process(13).

For this purpose it is possible to use the well-known relations for the coefficients \( C_1, C_2, C_3, C_4, C_5 \), of linearised Equations (1)\(^{12} \)

\[ \begin{align*}
C_1 &= -\frac{C_{M_e}}{I_y} \frac{\rho}{2} \frac{V}{S} \tau^2 \left[ 1 \right], \\
C_2 &= -\frac{C_{M_e}}{I_y} \frac{\tilde{\tau}}{2} \left[ 1 \right], \\
C_3 &= -\frac{C_{M_e}}{I_y} \frac{\tilde{\tau}^2}{2} \left[ 1 \right], \\
C_4 &= -\frac{C_{M_e}}{\rho} \frac{V}{2} \left[ 1 \right], \\
C_5 &= -\frac{C_{M_e}}{\rho} \frac{V}{2} \tau \left[ 1 \right], \\
C_6 &= \frac{C_{M_e}}{\rho} \frac{V}{2} \tau^3 \left[ 1 \right], \\
C_7 &= \frac{C_{M_e}}{I_y} \frac{\tilde{\tau}}{2} \left[ s^{-1} \right], \\
C_8 &= \frac{C_{M_e}}{I_y} \frac{\tilde{\tau}^2}{2} \left[ s^{-1} \right].
\end{align*} \]  

and the expression for obtaining the gain of the pitch damper

\[ G_q = 2 \frac{\sqrt{C_1 + C_3 + C_5}}{C_3} \ldots (13) \]

With Equation (10) solved, the coefficients (12) of the linearised equations for the system 'model-damper' will be:

\[ C_{1M} = \frac{C_{M_e}}{I_{1N}} \frac{\rho N \sqrt{K_L}}{2} \ldots (14) \]

And from this:

\[ C_{2M} = \frac{1}{K_L} C_{2N}; \quad C_{3M} = \frac{1}{K_L} C_{3N}; \quad \ldots (15) \]

\[ C_{4M} = \frac{1}{K_L} C_{4N}; \quad C_{5M} = \frac{1}{K_L} C_{5N}. \]

Therefore, with this equation, the gain of the pitch damper for the DSM will be:

\[ G_{q_{ph}} = \frac{2 \frac{1}{K_L} C_{2N} \frac{1}{\sqrt{K_L}} C_{1M} \frac{1}{\sqrt{K_L}} C_{4N}}{1 C_{3N} \ldots (16) \}

\[ \times \frac{1 \frac{1}{K_L} C_{4N} \frac{1}{\sqrt{K_L}} C_{3M} \frac{1}{\sqrt{K_L}} C_{5N}}{1 C_{5N}}; \]

\[ G_{q_{ph}} = \sqrt{K_l} G_{q_{ph}}; \]

Analogously, the transformation for roll and yaw dampers, have the same relation between the gain of the aircraft and that of the DSM.

If take the control law for a longitudinal zero-constant-error system as follows\(^{15} \):

\[ \ddot{a} = G_q \cdot \dot{e} + G_{\beta} \cdot \dot{\beta} + G_{\phi} \cdot \dot{\phi} \int \rho dt \ldots (17) \]

where: dimension of gain;

\[ G_{\beta} \left[ \frac{\text{degree of elevator}}{\text{degree of pitch angle}} \right] \]

is the stabiliser deviation on one degree with aeroplane deviation on one degree on pitch angle, and dimension of factor;

\[ G_{\phi} \left[ \frac{\text{degree of elevator /s}}{\text{degree of pitch angle}} \right] \]

is the stabiliser deviation with a speed of one degree per second with aircraft deviation on one degree on pitch angle (factor \( G_{\phi} \) defines the constant of integration, or speed of the stabiliser deviation, in the integral control law) and making an analogous transformation, then obtain;

\[ G_{q_{ph}} = \sqrt{K_L} G_{q_{ph}}; \quad G_{\beta_{ph}} = G_{\beta_{ph}} \ldots (18) \]

\[ G_{f_{ph}} = \frac{1}{\sqrt{K_L}} G_{f_{ph}}. \]
In this case the scale coefficients obtained, by the first (expression 11) and second (expression 15 and 18) modes, coincide. The validity of this ratio ensures the fulfillment of equalities\(^8,9\), i.e. it makes the products of the scale coefficients equal to the unit. However, following the order, it means the existence of the dimensionless similarity criterion, which must be numerically equivalent for similar phenomena. Therefore, it is possible to consider that the presence of necessary similarity conditions for the FCS of the aircraft and that of the DSM has been proven.

It is evident that the value and dimensions of the gains are determined by means of coefficients from the equations for the system ‘Aircraft-FCS’. Since these coefficients for the DSM and for the aircraft have a certain relationship, therefore after all transformations in the equations from aircraft to model’s gains they remain only as a matter of scale, which are defined by dimensions of its gains. Therefore, in all cases, the coefficients to transfer from the aircraft’s to the model’s gains, depend only on the actual dimensions of both, and are defined by the following equations:

\[
K = 1, \text{ in the time dimension } \left[ \text{degree} \right] \\
K = \sqrt{K_1}, \text{ in the time dimension } \left[ \text{degree} \right] \\
K = 1/\sqrt{K_1}, \text{ in the time dimension } \left[ \text{degree} \right]
\] . . . (19)

5.0 SUFFICIENCY CONDITIONS

To ensure sufficiency of conditions, at first the DSM must be manufactured as an exact copy of the real aircraft, but on a smaller scale, this provides the first condition of sufficiency, that is, geometrical parameter similarity between the model and the real object. Second, it is necessary to provide a functional similarity of the block diagram of the FCS, i.e. to keep in the DSM formulation the same control laws as on the real aircraft; which is not usually a problem. For the model, the precise same view of the control law is selected. In the case of the FCS of the designed DSM, its control laws are not arbitrarily selected, but are taken directly from the real aircraft.

The numerical values of DSM FCS gains are received from the appropriate gains of the aeroplane FCS by means of Equations (18). The physical adjustment of gains is made by means of the model autopilot programming (or by adjustment of the electrical parameters of the autopilot) in the same way as setting-up of gains on an aeroplane.

Third, ensuring similarity of the starting boundary conditions of confronted phenomenon: the flight of the real object and its model occur in a real atmosphere and at a definite altitude h. It is also necessary for meeting similarity to comply with the next relationship:

\[
K_h = \frac{\tilde{h}_M}{\tilde{h}_N} = \frac{\tilde{h}_{AM}}{\tilde{h}_{AN}}
\] . . . (20)

\[
\tilde{h}_M, \tilde{h}_{AM}, \tilde{h}_N, \tilde{h}_{AN}
\] . . . (21)

If in the manufacturing of the model, it is not possible to provide equality of density coefficients i.e.:

\[
\tilde{h}_M = \tilde{K} \tilde{h}_N (\text{where } K \neq 1)
\] . . . (22)

then, to comply with Equation (18), it is necessary to ensure,

\[
\tilde{h}_{AM} = K \cdot \tilde{h}_{AN}
\] . . . (23)

This is provided at the expense of adjusting the test flight altitude of the model. For example, the flight of an aircraft at 8,000m (\(p = 0.536\)) may be simulated with a flight of the DSM at 5,000m (\(p = 0.751\)).

These are in essence the existing necessary and sufficient conditions of similarity for the simulation of the FCS of the aircraft by means of Dynamically Similar Models.

6.0 CONCLUSIONS

The physical modelling of a FCS by means of large scale dynamically similar free-flying models has a long-range perspective, which allow for a significant increase in the quality of dynamic development of new types of aircraft, and their FCS; with the advantage of drawing nearer the physical model to the natural object being simulated.

Using two methods for proving the existence of necessary similarity conditions, on the one hand the formal mathematical method, and on the other dealing with the physical sense of the phenomenon, we increase the reliability of the obtained results.

Such results show the possibility of physically modelling the dynamics of flight, taking into account all peculiarities of aerodynamics, block diagram equivalence of the FCS, as well as real atmospheric conditions. Scale coefficients for transitioning from the FCS’s aircraft gain factors, to those of the FCS of the DSM allow us to simplify the development process of the DSM and its FCS, because of the exclusion of some traditional stages in design. Furthermore, it allows us to transfer more easily the flight-test data from the DSM’s FCS to the aircraft FCS.

Finally, it is necessary to note that the question of similarity of servos and the qualitative characteristic of the FCS still demand further investigation.

REFERENCES:
