High-accuracy four-dimensional trajectory prediction for civil aircraft

W. Schuster  M. Porretta  W. Ochieng
w.schuster@imperial.ac.uk
Centre for Transport Studies
Imperial College
London, UK

ABSTRACT

Current state-of-the-art trajectory prediction tools typically model aircraft as three-dimensional point-masses, and make a number of simplifying assumptions about the actual and anticipated dynamics states of the aircraft. They are typically based on predefined settings obtained from existing databases such as EUROCONTROL’s BADA rather than real-time information, including on the environment, available onboard the aircraft. This significantly limits trajectory prediction performance. This paper proposes a high-accuracy four-dimensional trajectory prediction model for use onboard civil aircraft, as well as by ground-based systems, which addresses these limitations. It is designed for strategic traffic capacity optimisation and conflict-detection and resolution over time-horizons covering the entire duration of a flight. The model incorporates a number of features including a novel flight-control-system and an enhanced flight-script that incorporates new taxonomy and content thereby enabling better definition of aircraft intent. The accuracy of the model is characterised using operational data acquired during a real flight trial. Results show that the performance of the proposed model is significantly better than the current models. Its accuracy is better than the required navigation performance for departure, en route and Non-Precision-Approach phases of flight.
1.0 INTRODUCTION

European skies are congested and current airspace capacity is reaching its limit (SESAR\(^1\)). The demand for air travel continues to rise with the consequence that traditional Air Traffic Management (ATM) methods do not satisfy capacity demands, resulting in delays and associated negative impacts on the economy, safety and the environment. Therefore, there is an urgent need to develop novel methods for ATM, making use of current and future technologies to increase capacity and efficiency without jeopardising safety and the environment.

The Single-European-Sky ATM Research (SESAR\(^1\)) and Next Generation Air Transportation System (RTCA\(^2\)) initiatives recognise that at the core of a more efficient navigation is the need to guarantee common situational awareness between all traffic and Air Traffic Control (ATC), and improve the ability to visualise the evolution of air traffic as a function of time. Key to both is the ability to accurately and reliably predict 4D aircraft trajectories and distribute these between all the relevant stakeholders, including ATC and surrounding traffic. Therefore, the performance and reliability of Trajectory Prediction (TP) will be a major factor in future ATM performance. TP is expected to be at the core of future ATC Decision Support Tools (DSTs) including Conflict Detection and Resolution (CDR) tools, and Airborne Self-separation Assistance Systems (ASAS). High-performance TP has the potential to reduce controller workload and increase airspace capacity, by improving the capability of ATC to perform the necessary synchronisation and separation activities in advance. This should enable the detection of conflicts earlier, making it possible to provide an optimised strategic approach to conflict resolution instead of the current inefficient tactical process.

Conceptually, TP consists of a number of functions: characterisation of the aircraft intent and environmental conditions, modelling of the aircraft dynamics and modelling of the flight-management system (FMS). The design of a TP model is determined by its operational application. Whilst some TP models may be suitable for short-duration (e.g. a few minutes) applications, such as tactical conflict detection and resolution, others may be designed for medium-duration (e.g. half an hour) and long-duration (e.g. several hours) applications, such as strategic traffic-flow optimisation. Therefore, a key parameter of interest for TP is the Look-Ahead Time (LAT). Ideally, a TP model should be able to realistically predict aircraft trajectories from the departure gate to the arrival gate in advance. This would maximise the benefits provided by TP, by enabling the identification of conflicts well in advance of the occurrence and thereby the strategic optimisation of air traffic capacity. At the same time, long-duration TP also enables short- and medium-duration conflict detection and resolution.

Models such as TSAFE described in Paielli\(^3\) and Erzberger\(^4\) were designed for short-duration TP and are therefore not suitable for strategic approaches to ATM. Other models, such as Glover\(^5\) and Fairley\(^6\) were designed for longer LATs, with the potential for strategic ATM provision. However, they make a number of simplifying assumptions about the mathematical models underlying aircraft dynamics. In particular, the actual and anticipated states of the aircraft are modelled based on predefined settings obtained from the EUROCONTROL BADA data set (EUROCONTROL\(^7\)) rather than making maximum use of intent information available onboard the aircraft. Moreover, environmental constraints are not realistically taken into consideration. Porretta et al\(^9\) developed an enhanced TP model, designed for long-duration LATs, which addressed some of the limitations of the models in the public domain, by incorporating the following two new features:

- The information about the Expected Time of Arrival (ETA) was considered in the evaluation of the True Air Speed (TAS) to be tracked by an aircraft during the flight. Whilst previous models used the thrust settings for a given phase of operation and accepted the resulting
‘nominal’ speed, using the ETA allowed a more accurate estimation of the speed requirements for the specified flight leg.

- Aircraft dynamics and operational limitations were considered for lateral guidance and subsequent bank-angle requirements determination.

However, a number of limitations remain that should be resolved in order to further improve the performance of TP. With respect to the state-of-the-art:

- The nominal speed requirement computed by the speed management unit is based on the assumption of an average aircraft speed to be achieved for a given flight-segment. In practice, this would typically result in an instantaneous and step-wise speed change requirement at the beginning of each segment. This can produce a nominal speed profile that is unrealistic and different from the actual speed profile of the aircraft, with the consequence of potentially significant along-track errors.

- The lateral controller does not consider the effects of the lateral bias-component of the wind. This results in an offset of the predicted trajectory from the actual trajectory.

- Thrust settings are based on the EUROCONTROL BADA inputs. However, these settings are not always representative of actual thrust settings used by the aircraft, especially during the climb- and descent-phases of flight, resulting in significant trajectory prediction errors.

- The flight-path angle is computed using an assumed Energy Share Factor (ESF) provided by the BADA. This ESF specifies how much of the available power is allocated to the change in height and to the change in speed. However, the values in the BADA do not consider the actual state of the aircraft or the required allocation of thrust to the rate-of-change of height and to the change in speed of the aircraft in order to follow the intended flight-path.

This paper builds on the basic framework of the model by Porretta et al. and addresses the limitations above to realise a high-accuracy and robust TP model that makes maximum use of aircraft actual state and intent information available from onboard systems. The TP model is intended to operate both onboard aircraft as well as in ground-based systems and enables the prediction of trajectories over the duration of an entire flight. To better determine the nominal aircraft speed profile, the model in this paper develops a new speed management unit that accounts for gradual nominal acceleration requirements, with speed constraints (introduced into the revised flight-plan) at the beginning of each segment, and takes into consideration the surrounding wind-field. Wind-correction factors are introduced into the heading management unit and the modelling of aircraft dynamics, for a more realistic lateral control. A new thrust controller makes use of aircraft acceleration, speed and Rate-Of-Climb-Descent (ROCD) requirements, to better represent actual thrust requirements, eliminating the need to rely on potentially non-representative predefined settings. Using aircraft speed and ROCD requirements, a new Flight Path Angle (FPA) controller is developed to eliminate the need to rely upon predefined ESF settings. Each of these novelties is discussed in turn. The paper tests and characterises the accuracy of the model when applied to a typical commercial aircraft flight, before concluding in Section 5.

2.0 CHARACTERISATION OF AIRCRAFT INTENT

Aircraft intent is defined in this paper as a structured set of instructions describing unambiguously the intended operational state of the aircraft over a given time-horizon. This set of instructions
determines the evolution of the dynamic state of the aircraft, computed by a ‘Trajectory Prediction’ (TP) module. The basic assumption is made that the aircraft follows each instruction to within the limits of its operational capability, taking into consideration surrounding environmental factors, such as wind. Each instruction is defined by a start-time and an end-time, thereby specifying the length of time during which a given instruction is to be executed. A set of instructions forms an operation, which unambiguously determines the aircraft motion during a given time interval. The point in space and time at which a change in such a set of instructions occurs is referred to as a Trajectory Change Point (TCP) and the time-ordered sequence of such TCPs defines the intended aircraft trajectory, i.e. the flight-script (FS).

In the TP model proposed in this paper, each TCP is defined by the following variables:

- A Way Point (WP), defined by horizontal and vertical position in terms of latitude, longitude and height.
- A Required Time of Overfly (RTO), describing the exact time at which the aircraft is required to fly-over or fly-by the TCP. The conceptual difference with respect to the ETA is that the latter corresponds to the time at which the aircraft is expected at the next TCP given that it makes no special attempt to reach it at a given time. In the RTO case, the aircraft adapts its configuration in order to reach the TCP at the specific time provided in the FS.
- A Required Initial Ground Speed (RIGS), describing the speed at which the aircraft is required to fly at the TCP. This is to ensure reliability and robustness of the TP as described in Section 3.4.1.
- Turn-Modes (TM), describing whether the aircraft is expected to ‘fly-over’ or ‘fly-by’ the TCP.
- Holding Modes (HM), describing whether the TCP belongs to a ‘holding area’.
- Expedited Descent Modes (EDM), describing the spoiler configuration from the current until the next TCP.

Of particular note is the inclusion of the ‘Required Initial Ground Speed’ (RIGS) for each TCP in the flight-plan. The FS should be defined such that for each segment the aircraft configuration is unambiguous. In order to optimise the performance of TP, this paper allocates TCPs for each point in space at which any of the following changes in configurations takes place: aircraft aero-dynamic configuration, non-linear variation in ground-speed (or preferably true airspeed), nominal heading, nominal flight-path-angle, thrust settings, environmental factors and reference frame. The more of these criteria are taken into consideration, the more accurate is the FS representation of the true trajectory, and hence the more accurate the TP process.

## 3.0 TRAJECTORY PREDICTION MODEL

The basic framework of the TP model proposed in this paper is summarised in Fig. 1. The model consists of three high-level units: Input Data (ID), Aircraft Dynamics System (ADS) and Flight Management System (FMS). These units are discussed in the following sections.

### 3.1 Input data

The Aircraft Dynamics System (ADS) requires an initial input vector of data that defines the initial state of the aircraft at the time at which TP commences. The data consists of the initial time, aircraft position, true-airspeed, magnetic heading, bank-angle and flight-path-angle. Each can be obtained from onboard sensors. Furthermore, the initial mass is required and can be ob-
tained from the take-off mass and the fuel-consumption rate associated with prior aircraft thrust settings. The fuel-flow rate is obtained from aircraft performance data available in BADA (EUROCONTROL). Additionally the specification of the spoiler, flap and landing gear configurations, available from the FMS, is required.

A key input to the ADS and the FMS is the wind field, which can be divided into bias and stochastic components. The bias component is obtained using either information available from the nearest (in both position and time) wind-field measurements or forecasts, such as the meteorological data provided by ATC. Alternatively, past wind-field measurements made onboard the aircraft can be used. The latter approach has the advantage that real onboard measurements provide in general a more accurate measurement of the actual wind-field than wind-forecasts. Various models have been proposed for the stochastic component, and typically assume a random field with a Gaussian distribution (Glover et al. (5,10)). However, no conclusive experimental evidence has been provided to support this assumption. Further work is thus required to demonstrate the validity of such models. A preliminary study carried out in this paper has shown that a Gaussian distributed wind-field (28kt, 95%) causes a degradation in the TP accuracy at the level of 10%. However, more detailed studies are required to confirm these results. As a result, the stochastic component is currently set to zero in the TP model developed in this paper.

Further inputs required by the ADS are Aircraft Performance Data (APD), specifying aircraft operational characteristics. Typical parameters include aircraft type, mass information, parameters related to the aircraft flight envelope and aerodynamics, as well as thrust and fuel consumption settings. These parameters can be obtained from suitable APD sets, such as EUROCONTROL’s BADA. Additionally, since aircraft nominal performance varies with the surrounding state of the atmosphere (e.g. temperature and pressure), an atmospheric model is required. This paper adopts the International Standard Atmosphere (ISA) model developed by the International Civil Aviation Organisation (ICAO).
3.2 Aircraft dynamics system

Aircraft motion is modelled using a 6 Degree-of-Freedom (DOF) Point-Mass-Model (PMM), derived from basic aerodynamics and Newtonian laws. Civil aircraft operate near trimmed flight conditions at all times, allowing aircraft motion to be described by a non-linear control system (Glover et al.5,10) with six state-variables: the 3-dimensional (3D) position \((x, y, h)\), the aircraft true airspeed \(V_{TAS}\), the aircraft heading angle \(\Psi\), defined with respect to the east-direction positive counter-clockwise, and the aircraft mass \(m\).

Aircraft dynamics is controlled by the following variables: engine thrust \(T\), bank angle \(\phi\), Flight-Path Angle (FPA) \(\gamma\) and drag coefficient \(C_D\). These emulate practical controls available to pilots who, for example, control the drag by changing the aircraft configuration, including flaps, slats, spoilers and landing gear. Additionally, aircraft dynamics is perturbed by the surrounding wind-field, represented by a 3D input vector \((w_x, w_y, w_v)\), describing the wind-field components along the \(x, y\) and vertical directions.

Combining the state-parameters with the aircraft dynamics control and perturbation parameters, the aircraft motion in this paper is described by the following non-linear control system:

\[
\dot{x} = V_{TAS} \cos \Psi \cos \gamma + w_x
\]

\[
\dot{y} = V_{TAS} \sin \Psi \cos \gamma + w_y
\]

\[
\dot{h} = V_{TAS} \sin \gamma + w_z
\]

\[
\dot{V}_{TAS} = \frac{T}{m} \cos \epsilon + \left( \frac{D}{m} + g \sin \gamma \right) = \frac{T}{m} \cos \epsilon - \left( \frac{C_D S p}{2} \frac{V_{TAS}^2}{m} + g \sin \gamma \right)
\]

\[
\dot{\Psi} = \frac{L}{m V_{TAS}^2} \sin \phi + \tau T + W_{xy} = \frac{C_L S p}{2} \frac{V_{TAS}}{m} \sin \phi + \frac{\sin(\gamma + \epsilon)}{m V_{TAS}^2} T + W_{xy}
\]

\[
\dot{m} = -\eta T
\]

In Equation (1), the ‘dot’ above each symbol indicates the first derivative with respect to time. \(D\) is the total drag, \(S\) is the total wing surface area obtained from BADA, \(\rho\) the air density computed using the ISA model and \(g\) the gravitational constant set to 9.80665m/s\(^2\). Furthermore, \(L\) is the lift, \(C_L\) the aerodynamic lift coefficient, \(\tau\) a parameter taking into consideration a small vertical component of the thrust that exists even under level-flight conditions, \(\epsilon\) the thrust-offset angle with respect to the wing-chord (as shown in Fig. 2), \(W_{xy}\) is the impact of the wind-force acting perpendicular to the flight-direction upon the rate of heading change, and \(\eta\) a factor describing the fuel consumption rate as a function of thrust, obtained from BADA. In this paper, \(W_{xy}\) is computed as:

\[
W_{xy} = \frac{\rho (S \sin \phi + S_{body/lat})(w_x^2 + w_y^2)}{V_{TAS} m \cos \gamma} \alpha \sin \left( \psi - \arctan \left( \frac{w_y}{w_x} \right) \right)
\]

\[
\ldots (2)
\]
where, $s_{\text{body/lat}}$ is the lateral surface area of the aircraft body, and

$$\alpha = \begin{cases} -1 \\ +1 \end{cases}$$

depending on whether the aircraft turns into the wind or away from it respectively. The lateral aircraft body surface-area is not included in BADA, and is therefore estimated in the current implementation of the model using the following formulation:

$$s_{\text{body/lat}} = \Delta \cdot \Lambda$$

where $\Delta$ is the aircraft diameter and $\Lambda$ is the aircraft length.

The thrust-offset angle is currently set to zero until these data become available in BADA. However, since this angle is typically very small, neglecting it is not expected to have any significant impact upon TP performance. Under trimmed flight conditions, the vertical lift balances the aircraft weight, including a correction term for the thrust-angle offset, and is derived as:

$$C_L = \frac{2\left(mg - T\sin(\gamma + \epsilon)\cos \phi\right)}{\rho V_{\text{CAS}}^2 S \cos \gamma \cos \phi} \quad \ldots (3)$$

Aircraft operational limitations are emulated by applying a constraint to the state, control and input variables of the model. These limitations are specific to each aircraft type and are derived from the corresponding parameters within the BADA. Finally, it should be noted that in the current implementation of the model, the effects of the vertical wind-gradient are not taken into consideration. Work is in progress to include these effects in a future version of the model. However performance benefits are expected to be small.

### 3.3 Flight management system

The FMS is a control system which emulates typical (auto-) pilot procedures. It uses a set of instructions to compute the required flight control parameters (as described in Section 3.2) based on the aircraft instantaneous state as obtained from the ADS and the intent information obtained from the Flight Script (FS). These instructions are formulated by the Flight Operation Mode System (FOMS) as a set of discrete variables describing the instantaneous operating mode of the aircraft. In the current TP model, each operating mode is described by a combination of eight

3.3.1 Flight Operation Mode System (FOMS)

The discrete state for the FMS described in this section is similar to that proposed in Porretta et al. However, this paper introduces a new method for the determination of the transition point between TCPs that is more representative of operational procedures. Furthermore, the condition placed on aircraft height by Porretta et al. for the update of the TCP has been removed, as it is not in line with operational procedures. In the implementation of the TP model in this paper, a distinction is made between fly-by and fly-over modes in the determination of the transition point between TCPs. For the fly-over scenario, a new update condition that is representative of operational procedures has been implemented. As shown in Fig. 3, a given TCP is deemed to have been reached if the aircraft (indicated by the ‘star’) has crossed the perpendicular to the median of the angle subtending the current and next segments. This condition is satisfied if

\[
\sin\left(\frac{\Delta \psi - \pi}{2} - \omega\right) \geq 0
\]  

In the case of ‘fly-by’ turn modes, the aircraft follows the arc at \( TCP_{i+1} \) as shown in Fig. 4. In order to provide for turn-anticipation, as required by fly-by turns, a threshold is computed to determine the ‘anticipation-distance’. This distance is a function of the turn-radius under nominal bank-angle conditions at \( TCP_{i+1} \) and is computed using Newtonian equations of motion as:

\[
\delta = \frac{V_{TAS}^2}{g\tan(\psi_{target})} \tan\left(\frac{|\Delta \psi|}{2}\right)
\]  

where \( \psi_{target} \) is the target angle-of-bank. Therefore, the aircraft should initiate a turn at a distance \( d \) from the last TCP (TCP) such that the following condition is satisfied (see Fig. 4):

\[
d > d_i - \delta
\]

As soon as this condition is satisfied, the TCP is updated and the aircraft is deemed to be on course to \( TCP_{i+2} \).

3.3.2 Flight Control System (FCS)

The FCS, part of the FMS proposed in this paper, uses four control variables as described in Section 3.2, i.e. thrust, bank-angle, flight-path-angle and drag coefficient. Each is adjusted considering the aircraft instantaneous state, FOMS instructions and FS information, with the goal to minimise deviations from the intended 4D trajectory described in the FS. Previous models have adopted various strategies in relation to controlling the four variables. A key limitation of these approaches is that they often use predefined recommended settings provided by BADA, without accounting for instantaneous aircraft state or intent. Hence a new FCS is proposed which includes new thrust and flight-path-angle controllers, and an enhanced lateral controller. In comparison to
previous models, these controllers more accurately emulate actual aircraft dynamics and operational procedures. This is in line with the overall approach in this paper to minimise reliance on pre-defined parameter settings and make maximum use of aircraft intent information. The FCS makes use of the required rates of vertical changes and speed changes to determine thrust and flight-path angle settings, as described below.

**Thrust controller**

In Porretta et al\(^{(8)}\), for thrust, nominal values are set to balance the drag acting on the aircraft during level-flight, while for climb and descent, predefined fixed values are set based on BADA recommendations, accepting the ROCD that is obtained as a result of these settings. The drawback of this approach is that neither the instantaneous ROCD nor the acceleration required to follow the intended aircraft trajectory are considered. Therefore, in the proposed model, the thrust of the aircraft is set according to the climb-mode and the acceleration-mode. During climb and descent, the thrust is determined based on both flight-path-angle and acceleration requirements. Newtonian laws of motion enable the derivation of the required thrust as follows:

\[
T = T_{\text{cruise}} + mg \sin \gamma + ma
\]

where \(\gamma\) is the required flight-path-angle, \('a'\) the rate-of-change of speed and \(T_{\text{cruise}}\) the thrust under level-flight and constant speed conditions, essentially determined by the drag acting on the aircraft:

\[
T_{\text{cruise}} = \frac{C_D \rho S V_{\text{TAS}}^2}{2}
\]

In Equation (7), both \(\gamma\) and \('a'\) can be positive or negative and are computed from aircraft TAS and ROCD requirements (see Section 3.4). It should be noted that thrust settings during climb and descent determine the ROCD, while thrust controls the TAS in level-flight conditions when \(\gamma\) is approximately zero.

**Flight-path-angle controller**

The model proposed in Porretta et al\(^{(8)}\) computes the flight-path angle using an assumed Energy Share Factor (ESF) recommended by BADA. The ESF is introduced in BADA to specify how
much of the available power is allocated to the change in height and to the change in speed\(^7\). However, the values provided by BADA do not consider the actual state of the aircraft or the required allocation of thrust to the rate-of-change of height and to the change in speed to actually adhere to the intended flight-path. For this reason, a more realistic FPA is determined in this paper based on both the ROCD and the speed requirements. These requirements are determined from the aircraft instantaneous state and the next target TCP state-requirements obtained from the FS. The FPA \(\gamma\) is then determined as:

\[
\gamma = \sin^{-1}\left(\frac{\text{ROCD}}{v_{GS}}\right) \quad \ldots (9)
\]

where \(v_{GS}\) is the ground speed. During the climb and descent phases, the FPA controls the aircraft TAS. During level-flight, the FPA controls the ROCD required to correct any small deviations from the nominal altitude as a result of, for example, a variation in the vertical wind-gradients.

**Bank-angle controller**

Lateral guidance is needed to correct lateral offsets from the required trajectory (referred to as Cross-Track Errors or CTE) and Heading Errors (HE). It is assumed to be provided by adjusting exclusively the bank-angle, which controls the heading of the aircraft. Neglecting the thrust-offset angle in Equation (1), the change in heading is computed as:

\[
\Psi = \left(\frac{C_a S \rho V_{TAS}^2}{2 \ m + \frac{\tan \gamma}{m V_{TAS}} T}\right) \sin \Phi + W_y \quad \ldots (10)
\]

In turn, the heading angle controls the horizontal position of the aircraft through

\[
\begin{align*}
\dot{x} &= V_{TAS} \cos(\Psi) \cos(\gamma) + w_x \\
\dot{y} &= V_{TAS} \sin(\Psi) \cos(\gamma) + w_y
\end{align*} \quad \ldots (11)
\]

In order to emulate the aircraft control system for lateral guidance, Glover et al\(^5\) proposes a linear controller to determine the bank angle requirements as a function of both the CTE \(\delta\) and the HE \(\Delta \theta\):

\[
\phi = f[\delta(t), \Delta \theta(t)] = k_1 \delta(t) + k_2 \Delta \theta(t) \quad \ldots (12)
\]

A comprehensive study to determine the values of the constant gains \(k_1\) and \(k_2\) can be found in Glover et al\(^5\). In Porretta et al\(^8\), a new lateral controller is proposed, where the effects of both the CTE and the HE are combined into an overall effective heading error, and limitations due to aircraft performance are taken into account. Although this new controller significantly improves the TP accuracy over that in Glover et al\(^5\), it does not account for the impacts of wind. The result is a systematic and deterministic bias in the cross-track and heading errors associated with the predicted trajectory. Moreover, the gain-factor is taken for granted, resulting in an underestimation of the actual bank angle used during turns, as shown later in this paper.

In the implementation of the FCS in this paper, the bank-angle is determined using a linear controller, based on a strategy similar to Porretta et al\(^8\), but with two innovations: inclusion of the wind-factor in the computation of the turn-radius and of the heading error. In order to
minimise potential deviations from the intended trajectory, the controller initiates suitable corrections for both the CTE and the HE as soon as they are observed. The bank controller computes the bank-angle required to get the aircraft back on track at the appropriate heading. The required bank angle is a function of both the CTE (δ on Fig. 5) and the HE (Δθ).

The CTE is expressed as a function of the angle β(t). This angle, shown in Fig. 5 (adapted from Porretta et al\(^8\) with updated annotations), is defined as the difference between the nominal heading of the current segment and the vector pointing from the current aircraft location to point ‘I’. This latter is defined as the intersection of the current segment with the arc of circle of a nominal turn initiated by the aircraft from its current location \(A\) at its maximum angle of bank. In Porretta et al\(^8\), the angle β(t) was effectively interpreted as an additional heading error, simplified here as:

\[
β(t) = -\arcsin\left(\frac{δ(t)}{2R}\right)\tag{13}
\]

Assuming that aircraft height and TAS are constant during the turn, the turn radius is computed in this paper as:

\[
R = \frac{mV_{TAS}^2}{mg \tan(φ_{target}) + W_x}\tag{14}
\]

The term \(W_x\) is introduced here to account for the impact of the horizontal wind-force upon the turn-radius. For a constant aircraft height, it is related to \(W_{xy}\) as follows:

\[
W_x ≈ mV_{TAS}W_{xy}\tag{15}
\]

where all parameters have the same meaning as before. This paper defines the overall heading error as a pseudo-heading error, given by:

\[
pHE(t) = Δθ(t) + β(t)\tag{16}
\]

A linear controller is then used to minimise the overall \(pHE\), expressed mathematically as:

\[
φ = f[δ(t), Δθ(t)] = k \times pHE(t)\tag{17}
\]

The tuning procedure described in Section 4.1 shows that the constant gain factor \(k\) provides optimal results when set to 2.4.

A further enhancement to the bank-angle controller is the consideration of the wind-factor in the computation of the heading error \(Δθ\) (as described below). In Porretta et al\(^8\), this heading error is computed as the difference between the current heading \(θ_{TAS}\) and the heading associated with the current flight-segment \(θ_{i+1}\), computed from the FS:

\[
Δθ(t) = θ_{TAS} - θ_{i+1}\tag{18}
\]

In the presence of a bias in the lateral component of the wind, this will result in constant heading error and a CTE because the aircraft is constantly pushed off its intended track by the wind. In real-life, in the presence of lateral winds, the aircraft is required to compensate for the effect of the wind by turning its heading into the wind. This is illustrated in Fig. 6. Given a wind-field
vector $\omega_{xy}$, the TAS vector $V_{TAS}$ must be such that the vector resulting from the sum of the TAS and the wind-field is parallel to the direction vector defined by the current segment. Mathematically the target heading $\theta_{\text{target}}$ in the presence of a lateral wind-field can be expressed as:

$$\theta_{\text{target}} = \theta_{i+1} + \Delta \theta_w$$

(19)

The effective heading error $\Delta \theta(t)$ of the aircraft is then given by:

$$\Delta \theta(t) = \theta_{\text{TAS}} - \theta_{\text{target}} = \theta_{\text{TAS}} - (\theta_{i+1} + \Delta \theta_w)$$

(20)

This means that the aircraft is effectively flying side-ways, referred to as ‘crabbing’. Bank-angle settings provided by the controller are saturated between minimum and maximum permissible values, associated with the operating limits of the given aircraft type. Such saturation assures that the aircraft is operating within its capabilities.

**Coefficient of drag**

The coefficient of drag $C_D$ is specified in existing models as a function of the coefficient of lift $C_L$. Furthermore, the expressions recommended for the calculation of $C_D$ depend on the discrete variables which specify the phase of flight. For example, changes in the aircraft configuration during the descent alter the aerodynamic drag force, and are modelled using appropriate correction factors or incremental terms in the proposed expressions for $C_D$\(^{(7)}\). In the approach phase, additional terms account for approach-specific flap and landing-gear settings, both of which increase the drag. Similarly, when the use of spoilers is required by an expedited descent, an appropriate multiplication factor is used to boost the coefficient of drag\(^{(9)}\). This paper adopts the same drag-model as proposed in Glover et al\(^{(5)}\).
3.4 Speed and ROCD requirements

3.4.1 Speed target

Robust and accurate speed control is key to high-accuracy trajectory prediction since even relatively small speed inaccuracies can result in significant along-track errors over prolonged time horizons. Moreover, speed profiles, in order to be realistic, must not show any step-wise discontinuities. In other words, an aircraft cannot adjust its speed in a step-wise manner. The proposed scheme in Porretta et al. computes the average speed requirement for each segment. A key limitation of this approach is that it typically creates a step-function in the predicted TAS profile, with discontinuities between adjacent flight segments, and speed oscillations. Hence, this paper proposes a new strategy for the determination of nominal TAS, taking into account aircraft operational procedures and limitations. Besides nominal speed requirements, the strategy considers instantaneous acceleration requirements to meet the RTO as well as the RIGS at the next TCP, taking into consideration environmental factors such as wind. The evaluation process consists of two subsequent steps, presented below.

Computation of the distance to the next TCP

The first step in the determination of the required TAS is the computation of the distance from the instantaneous aircraft position to the target TCP, defined as the one being used in the computation of the speed requirements. It is complicated by turn-anticipation as the actual distance to any given TCP depends on the turn-mode and the change in heading between subsequent segments. Hence, several steps are required, starting with an initial estimate of the distance to the target TCP. This is computed as the sum of straight-line distances:

\[
d_{\text{estimate}} = d_{d(i+1)}(t) + \sum_{m=1}^{m}\ d_{m}
\]

where \(d_{d(i+1)}\) is the straight-line distance from the instantaneous aircraft position to the next TCP and \(d_{m}\) are the straight-line distances between \(TCP_{m}\) and \(TCP_{m+1}\). An estimate of the average speed required to reach the target TCP at \(RTO(TCP_{tgt})\) is then computed based on the time available to reach this target TCP:

\[
\langle v_{\text{estimate}} \rangle = \frac{d_{\text{estimate}}}{RTO(TCP_{tgt}) - t}
\]

This estimate is used in the computation of the nominal turn-radius to achieve the given change in heading between two subsequent trajectory segments using Equation (14) and replacing the \(V_{TAS}\) with \(\langle v_{\text{estimate}} \rangle\). This turn-radius is then used in the computation of an improved estimate of the actual distance to be flown by the aircraft to reach its next target.

With regards to the specific turn-modes, the fly-by mode should account for the fact that the aircraft anticipates the turn at \(TCP_{i+1}\), and follows the arc-of-circle (shown in Fig. 7) to join the next segment. Accordingly, the distance to \(TCP_{i+1}\) is computed as follows:

\[
d_{\text{eff}} = d_{t} - R_{t} \tan \left( \frac{\Delta \Psi_{i+1}}{2} \right) + R_{i+1} \left( \frac{\Delta \Psi_{i+1}}{2} \right)
\]
where $d_{i,\text{eff}}$ is shown as a bold line in Fig. 7 and $d_i$ is the total straight-line distance between TCP$_i$ and TCP$_{i+1}$. In the case of the fly-over mode, it is assumed that the aircraft uses common operational procedures and flies along the arc-of-circle shown in Fig. 8, using a nominal bank-angle.

The distance flown between TCP$_{i+1}$ and TCP$_{i+2}$ is then computed as:

$$d_{i+1,\text{eff}} = d_{i+1} - 2R_{i+1} \sin(\Delta \Psi_{i+1}) + 2R_{i+1} \Delta \Psi_{i+1}$$

An improved estimate of the actual distance flown by the aircraft is then given by:

$$d_{i,\text{tg}} = d_{i+1}(t) + \sum_{m=0}^{n-1} d_{m,\text{eff}}$$

These distances are used together with the RTOs to compute the speed requirements, as shown below.

**Computation of the instantaneous speed requirements**

The model developed in this paper evaluates the nominal TAS by using the rate-of-change of speed.
a(t) requirement to compute the ground speed requirement as follows:

\[ v_{GS}(t + \Delta t) = v_{GS}(t) + a(t)\delta \] \hspace{1cm} (26)

The rate-of-change of speed is computed based on the current speed, the distance \( d_{tgt} \) between the current position and the target TCP, and the time available to reach the target TCP at RTOtg:

\[ a(t) = 2 \left( \frac{d_{tgt}}{\Delta t^2} - \frac{v(t)}{\Delta t} \right) \] \hspace{1cm} (27)

A variation in the initial speed at any instant in time can result in significant variations in the estimates of the required \( a(t) \), with the consequence of significantly different speed profiles. The resulting oscillatory behaviour of the ground speed requirement is illustrated in Fig. 9. For example, all three speed profiles shown in this figure satisfy the average speed requirements for each of the segments, yet the overall 4D trajectory flown is significantly different. Since aircraft make smooth adjustments in their speed, it is essential that the speed at the end of a given segment corresponds to the speed requirement at the beginning of the following segment. This continuity can be assured by setting a requirement on the initial ground speed at the beginning of each segment. As a result, the RIGS is introduced into the FS, as described in Section 2.

By requiring a given initial ground speed for each TCP, the speed profile is significantly constrained. The required \( a(t) \) can be expressed either as a function of the speed requirements or as a function of the distance and time requirements as shown by the following two equations respectively:

\[ a(t) = \frac{RIGS(t_{r_1}) - v_{GS}(t)}{(t_{r_1} - t)} \] \hspace{1cm} (28)

and

\[ a(t) = 2 \left( \frac{d_{tgt}}{(t_{r_1} - t)^2} - \frac{v_{GS}(t)}{(t_{r_1} - t)} \right) \] \hspace{1cm} (29)

These two equations can be combined into one, to uniquely define the required rate-of-change of speed:
Note that the time-factor in the denominator has been eliminated in the computation of \( a(t) \). The ground speed requirements are then computed using Equation (26). This method is robust against variations in the initial speed \( v_{GS} \) as may occur for example, as a result of varying wind-fields. The true airspeed vector is then computed as the sum of the required ground-speed vector and the instantaneous wind-field vector:

\[
v_{\text{target}} = v_{GS} - W_{xy}
\]  

This speed is used as an input to the FCS which determines the appropriate thrust and flight-path-angle settings (see Section 3.3.2).

3.4.2 ROCD Target

Currently, BADA recommendations for both thrust and ESF settings are used to determine the flight-path angle. The model developed in this paper improves accuracy by using aircraft intent information to determine an instantaneous target ROCD which, in combination with \( v_{\text{target}} \) determines the appropriate thrust and flight-path-angle settings to be used throughout the flight (see Section 3.3.2). Here, the instantaneous target ROCD is computed from the difference between the height of the next TCP and the current height, and the time available to execute this change in height. Mathematically this is expressed as:

\[
ROCD = (h_{r+1} - h(t)) \frac{v_{GS}(t)}{d_{\Delta Iy}(t)}
\]  

Figure 9. Illustration of speed profiles as a function of initial speed.
4.0 PERFORMANCE CHARACTERISATION

The performance of the TP model has been validated using real flight trials in European Air-space. Detailed Flight Data Records (FDR) were provided by EUROCONTROL (EUROCONTROL, 2009(13)). The FDR include aircraft 4D position, attitude, thrust settings, speed (True and Calibrated Air Speeds, Ground Speeds and ROCD) and environmental conditions (wind-field, temperature and pressure) as a function of time. The FDR used to characterise the performance of the TP model are for a three-hour Boeing 737-500 aircraft flight on 16 November 2005 from Sofia International Airport in Bulgaria (ICAO Code: LBSF) to Sheremetyevo International Airport in Moscow (ICAO Code: UEUE). This is the same reference trajectory that was used to evaluate the performance of the model in Porretta et al (8).

All the necessary aircraft performance parameters were taken from the BADA set (EUROCONTROL, 2009(7)). Additionally, the aircraft diameter and length were 3.54m and 31.0m respectively, resulting in a total lateral aircraft-body surface area of 109.74m².

The proposed TP model requires as input the aircraft intent in the form of TCPs, with the associated RTOs and RIGSs. These are not directly available from the FDR, and were allocated through a backward analysis using the approach described in Section 2. The resulting FS is a time-ordered sequence of TCPs that accounts for the most significant changes in the aircraft configuration, such as heading, TAS, ROCD and in the environmental conditions, such as wind. In this paper, the wind-field is estimated using wind samples from weather models in space and time, typically available to pilots and ATC. Furthermore, the assumption is made that aircraft can, as an alternative or in addition, make use of real-time airborne-measured wind-field data. The advantage of the former approach is that if wind-field measurements from weather models are available at the beginning of the flight, prediction for the entire trajectory can be carried out in advance. However, weather model data may have relatively large uncertainties, hence limiting prediction accuracy. The use of wind-field measurements onboard aircraft typically improves measurement accuracy, with the draw-back however, that the update rate determines the prediction time-horizon.

Only the Free-Flight (FF) portion of the flight is considered in the performance assessment, corresponding to the flight-time above Flight Level (FL) 240. The FF portion involves 44 out of the 80 TCPs in the FS, with time duration of 2 hours, 5 minutes and 25 seconds, i.e. 7,525 seconds.

![Figure 10. Maximum CTE as a function of the gain-factor ‘k’ used in the computation of the bank-angle.](image-url)
4.1 Tuning of the controller used for lateral guidance

The linear controller (Equation (12)) used by Glover et al\(^5\) to emulate the aircraft lateral guidance control scheme was tuned using a Monte Carlo method, making simplifying assumptions about the wind-field and the statistical properties of the CTE. This led to the determination of the control gains \(k_1\) and \(k_2\) for the CTE and the HE, respectively. In Porretta et al\(^8\), the proposed linear controller (Equation (17)) for lateral guidance incorporates the effects of the CTE into the HE, selecting the same controller gain used by Glover et al\(^5\) for the HE, i.e. \(k = k_2 = 1.2\). Studies in this paper suggest that this gain factor is not always able to realistically emulate aircraft turn-radii that are required to follow the intended trajectory, especially in the presence of lateral wind. Therefore, a new method to evaluate the controller gain is proposed here that consists in comparing the statistics of the CTE for different values of the gain factor \(k\) (as shown in Fig. 10).

In the range of \(k = (1.0 \text{ to } 2.4)\), large variations in the maximum CTE are observed. This is the result of the inability of the aircraft to follow its intended trajectory since it is not ‘allowed’ to bank sufficiently. Upon reaching a value of \(k = 2.4\), the curve is approximately flat. The same behaviour is observed for the standard deviation and the mean CTE (only maxima are shown for the sake of clarity). The reason for this behaviour is that any further increase in the allowable maximum bank-angle does not improve the ability of the aircraft to follow its turn and as a result no increase in performance is expected. From an operational perspective the aircraft would use the minimum bank-angle that is required to maintain a given turn-radius in order not to ‘overshoot’ its trajectory and cause it to zigzag around its centreline. Therefore, a value of \(k = 2.4\) is adopted in this TP model. Such value is also realistic from an operational perspective, especially when turning into the wind.

4.2 TP model accuracy

The TP accuracy is evaluated in terms of Euclidean Errors (EE), defined as the 3D distance between the actual aircraft position and the nominal position for each point in time. The EE are then projected along-track and perpendicular to the nominal track in the horizontal and the vertical, to evaluate respectively the Along Track Errors (ATE), Cross-Track-Errors (CTE) and Altitude-Errors (AE). Furthermore, the accuracy of the aircraft in meeting the RTOs is evaluated as Time-of-Overfly Errors (TOE). These are computed as the difference between the nominal time and the actual time of the aircraft for each point along the trajectory.

Numerical results for the case of high wind-sampling (approximately one sample every 60 seconds during the en route phase of flight) are presented in Table 1 and the time-evolution of these

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Maximum of absolute values</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATE</td>
<td>255.6m</td>
<td>726.9m</td>
<td>291.8m</td>
</tr>
<tr>
<td>CTE</td>
<td>8.6m</td>
<td>331.5m</td>
<td>55.2m</td>
</tr>
<tr>
<td>AE</td>
<td>1.6m</td>
<td>116.0m</td>
<td>19.1m</td>
</tr>
<tr>
<td>TOE</td>
<td>1.2s</td>
<td>3.3s</td>
<td>1.3s</td>
</tr>
</tbody>
</table>
errors is shown in Fig. 11. As shown in Table 1, the CTE and AE of the predicted trajectory have a mean that is very close to zero. Overall, there are no significant deviations from this mean, except for the AE which show larger errors around \( t = 1,100s \) and \( t > 8,000s \). These two time-intervals correspond to the climb and descent phases. The errors occur on transitioning between climb-modes due to the lack of anticipation of the change in aircraft configuration between climb-modes in the current TP model. The ATE has the largest mean, with maximum values of \(~727m\). It should be noted that the majority of the contribution to the overall EE is the ATE, which depends upon a correct estimation of the required TAS, and is affected by the along-track wind-component.

Overall, there is very good agreement of the predicted trajectory with the true trajectory as shown in Figs 11 and 12: the standard deviation (95%) of the ATE is at the level of 584m, of the CTE at the level of 110m, and of the AE at the level of 38m. An interesting parameter is also the TOE, which is at a level of 2·6s (95%). The TOE is effectively a measure of the level of capability of the TP model to accurately predict the arrival time at a given TCP, an important consideration in the assessment of 4D conflict detection and resolution strategies where timing is a key factor.

The wind-data can be obtained either from wind-prediction models or from onboard aircraft measurements. For example, for the current FDR data, wind-field measurements are provided at 60 second intervals during the en route phase of flight (with increased sampling during the climb, descent and approach phases). Therefore, the time-horizon of the TP process, assuming that each sample of the FDR wind-data is used in the TP process, corresponds to 60 seconds during the en route phase as well.

In order to emulate larger Look-Ahead Times (LAT), the wind samples provided by the FDR were down-sampled by varying factors. The impact on the accuracy of TP is shown in Table 2. Wind-field uncertainties have the largest impact on the ATE and TOE, with relatively minor impacts upon CTE and AE. Any significant difference between the actual and estimated along-track

![Figure 11. Time-evolution ATE, CTE, AE and TOE.](image)
Table 2
Comparison of accuracy of the novel TP model as a function of varying look-ahead times

<table>
<thead>
<tr>
<th>TP LAT (en-route)</th>
<th>Error</th>
<th>1 min</th>
<th>5 min</th>
<th>10 min</th>
<th>15 min</th>
<th>20 min</th>
<th>30 min</th>
<th>60 min</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean [m]</td>
<td>256</td>
<td>267</td>
<td>151</td>
<td>260</td>
<td>873</td>
<td>1347</td>
<td>1041</td>
</tr>
<tr>
<td></td>
<td>Maximum of absolute value [m]</td>
<td>727</td>
<td>1171</td>
<td>1137</td>
<td>962</td>
<td>2058</td>
<td>2947</td>
<td>1811</td>
</tr>
<tr>
<td></td>
<td>Accuracy (95%) [m]</td>
<td>584</td>
<td>731</td>
<td>1043</td>
<td>652</td>
<td>1137</td>
<td>1212</td>
<td>982</td>
</tr>
<tr>
<td>ATE</td>
<td>Mean [m]</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>8</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Maximum of absolute value [m]</td>
<td>332</td>
<td>332</td>
<td>333</td>
<td>328</td>
<td>333</td>
<td>531</td>
<td>330</td>
</tr>
<tr>
<td></td>
<td>Accuracy (95%) [m]</td>
<td>110</td>
<td>106</td>
<td>106</td>
<td>106</td>
<td>112</td>
<td>122</td>
<td>113</td>
</tr>
<tr>
<td>CTE</td>
<td>Mean [m]</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Maximum of absolute value [m]</td>
<td>116</td>
<td>116</td>
<td>139</td>
<td>116</td>
<td>188</td>
<td>231</td>
<td>123</td>
</tr>
<tr>
<td></td>
<td>Accuracy (95%) [m]</td>
<td>38</td>
<td>37</td>
<td>43</td>
<td>38</td>
<td>45</td>
<td>47</td>
<td>42</td>
</tr>
<tr>
<td>AE</td>
<td>Mean [s]</td>
<td>1.2</td>
<td>1.5</td>
<td>3.2</td>
<td>1.2</td>
<td>4.0</td>
<td>6.2</td>
<td>4.8</td>
</tr>
<tr>
<td></td>
<td>Maximum of absolute value [s]</td>
<td>3.3</td>
<td>5.7</td>
<td>7.4</td>
<td>4.9</td>
<td>9.3</td>
<td>13.3</td>
<td>8.4</td>
</tr>
<tr>
<td></td>
<td>Accuracy (95%) [s]</td>
<td>2.6</td>
<td>3.8</td>
<td>3.5</td>
<td>3.0</td>
<td>5.2</td>
<td>5.5</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Figure 12. Histograms of ATE, CTE, AE and TOE.
wind-components leads to a significant difference in the estimation of the required TAS. Since ATE are integrated over time, even relatively small offsets in TAS can result in significant ATE over time. For CTE and AE, the impact is less significant, since the required true heading (not the tracked heading!) and the required altitude are well-known from the flight-script, and are both independent of the wind-field. Any impact of the lateral or vertical components of the wind-field is thus easily corrected by the FCS. Note that there is a spike in the CTE for a LAT = 30 minutes. This is attributed to the unfortunate choice in sampling where one of the wind-field measurements used by the aircraft corresponded to an extreme in the wind-field strength. Such scenarios can be avoided by always using an average wind-field sampled over a given time-period rather than using an instantaneous measurement provided by the aircraft onboard systems. Future work will implement such averaging.

The accuracy of the current model has been compared to the model in Porretta et al (8) – based on the same portion of the FF and a sampling interval of 60 seconds for the wind-field. The accuracy differences are highly significant, with improvements in the overall maximum prediction errors by over 95% (e.g. 726.9m versus 14,854m for the maximum of the absolute value of the ATE), with other prediction errors improved by a factor varying between 65% and 95%. Overall, the accuracy improvements can be attributed to a significant enhancement in the level of realism applied in the novel TP model proposed in this paper, with focus on emulating as much as possible practical aircraft operational procedures. Amongst others, the significant improvement in the maximum errors should be noted, with magnitudes below the 1km level. This reduction in maximum errors is especially relevant, as it defines the overall reliability of the TP model in being able to ‘guarantee’ a certain worst-case performance.

Irrespective of the LAT, all parameters are at a level that is significantly better than the currently allocated lateral navigation system performance requirements (Schuster et al (14)) for the en route airspace, the Terminal Manoeuvring Area (TMA), the Initial Approach (IA), Intermediate Approach (IMA) and Non-Precision Approach (NPA), as well as the departure even for relatively large time-horizons (e.g. 60 minutes). The CTE (95%) are smaller than the lateral accuracy requirements and the maximum CTE are smaller than the Alert Limit requirements. A comparison is shown in Table 3. The results for the IA, IMA, NPA and departure have been extrapolated, based upon the assumption that the wind-field sampling is finer during those phases of operation, hence the reduced time-horizon.
5.0 CONCLUSIONS

Two key factors are at the core of four dimensional (4D) Trajectory Prediction (TP): representation of aircraft intent and the TP model. TP is limited by the capability of the Flight Script (FS) to accurately represent the actual flight trajectories. Therefore, a realistic and unambiguous representation of aircraft intent is critical. In turn, the TP model must emulate as far as possible practical operational procedures in following the trajectory defined by the FS. In the context of these two factors, and considering the state-of-the-art, this paper has proposed a number of innovations. Specifically, the model in this paper developed an advanced flight control system for trajectory prediction, benefiting from novel controls of speed, thrust and flight-path-angle, as well as an enhanced lateral controller. These developments lead to the definition of a new FS, able to more realistically define the aircraft intent.

The new developments have been shown to result in significantly better TP compared to state-of-the-art models. Moreover, the performance of the model developed in this paper is significantly better than the instantaneous positioning requirements of aircraft for the en route sector, as well as for flight-phases up to and including NPA and departure. The ability to predict aircraft trajectories to this high level of performance is expected to have significant benefits. Advanced decision support tools based upon TP will reduce controller workload, one of the key factors limiting airspace capacity. Moreover improved TP will allow more advanced conflict detection and improved conflict resolution for onboard aircraft safety systems, thereby contributing towards improving the safety, efficiency and environmental impacts of air travel.

ACKNOWLEDGEMENTS

The authors would like to thankfully acknowledge the EUROCONTROL TP team for providing the reference validation data used in the analysis of this paper.

REFERENCES

7. RTCA (2009), NextGen Mid-Term Implementation Task Force, RTCA Task Force 5.
9. EUROCONTROL (2009b), Reference Validation Data Base, EUROCONTROL TP team website.