Advancement of aerofoil section dynamic stall synthesis methods for rotor design

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ABSTRACT
Dynamic stall is a complex process encountered by an aerofoil in the unsteady flow environment such as a helicopter rotor in forward flight as well as a fixed wing aircraft in manoeuvres and in other unsteady situations. The onset of dynamic stall effectively determines the flight envelope of the helicopter. Significant effort are being made to develop CFD to capture the dynamic stall behaviour, however traditional engineering models based on lifting line theory still offer fast turn-round and broad understanding required for the rotor design process. This paper describes a new engineering model for dynamic stall, developed originally for wind turbine application at a typical Mach number of 0.12. The new dynamic stall model, with a better definition of stall onset, is based on improvements made to Beddoes’ original trailing edge stall model. This paper will describe and demonstrate the improvements in identifying both the stall-onset and the pitching moment break at high pitch rates, when being applied to a generic rotor aerofoil RAE 9651 at M = 0.3. Further validation against oscillatory tests and other Mach numbers are still required. However the study has provided sufficient confidence for it to be employed in a rotor analysis code.

NOMENCLATURE
\[ a \quad \text{co-ordinate of pitching axis in terms of half chord } (a = x/b) \]
\[ b \quad \text{half chord } (b = c/2) \]
\[ c \quad \text{chord length} \]
\[ C_C \quad \text{chordwise force coefficient} \]
\[CD\] drag coefficient
\[C_m\] pitching moment coefficient
\[\Delta C_{mv}\] pitching moment due to vortex
\[C_N\] normal force coefficient
\[C_{N\text{min}}\] normal force coefficient local minimum before full re-attachment
\[\Delta C_{Nv}\] additional normal force due to vortex lift
\[C_p\] pressure coefficient
\[|C_p|\] rise pressure coefficient suction rise at the leading edge (normally 2.5\% chord)
\[f\] separation location in terms of chord length
\[f', f''\] delayed separation function of \( f \)
\[M\] Mach number
\[r\] reduced pitch rate \( (r = c\alpha/2V) \)
\[s, \tau\] non-dimensional time \( (s = \tau = 2Vt/c) \)
\[t\] time parameter
\[T_f\] delay constant for separation point due to dynamic effect
\[T_p\] delay constant of attaining same \( C_N \) in a delayed leading-edge-pressure under unsteady condition
\[T_t\] delay constant for reattachment process
\[T_v\] time constant of vortex travelling over chord
\[T_{yl}\] vortex passage time constant
\[T_{\alpha}\] delay time constant for angle-of-attack due to dynamic effect
\[V\] free stream velocity
\[V_x\] shape function of normal force due to vortex
\[w_x\] flow speed on the aerofoil surface perpendicular to chord
\[x\] co-ordinate along chord centred at half chord
\[x^*\] dimensionless value of \( x \) in terms of half-chord \( (x^* = x/b) \)
\[z_o\] flow displacement perpendicular to chord
\[\alpha\] angle-of-attack or incidence
\[\alpha_0\] angle-of-attack of zero normal force or mean angle-of-attack of oscillation of aerofoil
\[\alpha_1\] breakpoint of separation
\[\alpha_{cr}\] critical stall-onset angle-of-attack
\[\alpha_{ss}\] static stall-onset angle-of-attack
\[\alpha_{st}\] stall-onset angle-of-attack
\[\alpha_{st0}\] constant critical stall-onset angle-of-attack
\[\alpha_{min}\] angle-of-attack at \( C_{N\text{min}} \)
\[\alpha_{min0}\] angle-of-attack at \( C_{N\text{min}} \) for static test
\[\theta\] co-ordinate transformation, \( \theta = \cos^{-1}(x/b) \)
\[\kappa\] reduced frequency \( (\kappa = \omega c/2V) \)
\[\eta_x\] forcing for circulatory pitching moment
\[\eta_x\] forcing for circulatory normal force
\[\lambda\] coefficient of least square fit
\[\lambda_m\] forcing for non-circulatory pitching moment
\[\lambda_n\] forcing for non-circulatory normal force
\[\tau_v\] non-dimensional time during vortex passage
\[\omega\] circular frequency of oscillation of aerofoil
\[\Delta\] a step change in forcing or in time
1.0 INTRODUCTION

Dynamic stall is a term used to describe the stalling process of an aerofoil, or a lifting surface, during unsteady motion. These conditions are normally associated with pitching and plunging motions and are of great interest to helicopter rotor aerodynamicists, who require an assessment of rotor loads and performance over the entire flight envelope of the aircraft.

The flow around an aerofoil under dynamic conditions differs significantly from that of static aerofoil. Extra features include the delay of boundary layer separation, the formation and shedding of a concentrated vortex, and a delay in flow reattachment from a fully separated state. As a result, under dynamic conditions, the aerofoil can produce a much higher lift than that under static conditions. Far from improving flight performance increased dynamic lift can induce a large nose-down impulsive pitching moment, which is of structural and aeroelastic concern.

In general the dynamic flow field produces large hysteresis in both the lift and pitching moment characteristics of the aerofoil section. The pitching moment hysteresis can cause a negative damping when the aerofoil oscillates about the stalling angle causing the rotor blades to experience stall flutter. Stall flutter ultimately defines the flight envelope of the rotor and serious effort has been expended in developing aerofoil sections that can delay the stalling point as in the case of the RAE 9000-series aerofoils developed by Wilby(1,2) under the British Experimental Rotor Programme (BERP).

Dynamic stall is a very complicated phenomenon and as new aerofoil sections are introduced more features that need to be explained and modelled continue to be discovered. However a good summary of the of the principal features of the dynamic stalling process were presented by Leishman(3), shown in Fig. 1:

Figure 1. Schematic of dynamic stall process on an oscillating 2D aerofoil(3).
Under dynamic conditions, circulation is shed into the wake from the trailing edge of the aerofoil and its presence in the wake near the trailing edge as it is being convected away causes a reduction in the lift and the adverse pressure gradient in the pressure recovery zone of the aerofoil, compared to the steady case at the same angle-of-attack;

- By virtue of induced camber effects, a positive pitch rate further decreases the pressure and pressure gradients at the leading edge for a given value of lift;

- In response to the external pressure gradients, additional unsteady effects occur within the boundary layer, including the existence of flow reversals without any significant separation.

To model the dynamic stall phenomenon, all these effects could be modelled by delays in the boundary-layer separation. The classical examples can be found in the first generation dynamic-stall model (FGM) by Beddoes(4), the second generation dynamic stall model by Leishman-Beddoes (L-B)(5), and more recently in an improved model of Sheng et al(6). These effects are associated with boundary conditions that govern the attached flow about the aerofoil but other important phenomena must also be included:

1) When an aerofoil is rapidly pitching up, the flow will remain attached even when the angle-of-attack is significantly larger than the static stall angle, and a large pressure suction in the leading edge region has been built up. The pressure suction is so strong that when separation finally occurs a concentrated vortex is formed near the leading edge and then transits over the aerofoil upper surface. It induces additional lift that could be modelled by a further delay in boundary layer separation, allowing lift further to increase(6). Once the vortex is shed into wake, the flow over aerofoil upper surface is fully separated and the extra lift disappears.

2) When the aerofoil is pitching nose down, the re-establishment of fully attached flow is delayed. Only when the angle-of-attack is well below the static stall angle, does the surface flow reach a fully attached state. Sheng et al(7) showed that the return from fully separated state comprises of two successive events: convection of stalled flow into the wake of the aerofoil downstream and the subsequent re-establishment of the boundary layer.

To provide some physical understanding of how improvements can be made, a number of issues will be addressed.

2.0 INDICATION OF STALL ONSET

Stall onset is a very important condition for the aerofoil operating in dynamic conditions, since it represents the limiting case of maximum lift with no penalty in $C_m$ and $C_d$ and it determines the flight envelope of helicopter rotors (McCroskey et al(8)). Identification of the onset of stall is very important for the design of rotors. Historically, several criteria have been employed to identify stall onset from experimental time histories produced by dynamic aerofoil section tests. Some of these criteria are illustrated in Fig. 2.

$C_n$ deviation/$C_n$ maximum:

Under dynamic conditions, normal force may maintain a linear-like increase to an angle-of-attack much larger than the static stall angle because of the delay of boundary-layer separation. For some aerofoils, before dynamic stall occurs, trailing-edge separation may be involved. This can be seen from the normal force curve, when the rate of increase of lift with incidence begins to fall away. However, when a concentrated vortex develops, the extra lift it induces may mask
the trailing-edge separation effect. In some cases, the vortex is so strong that an additional normal force can also be seen, when it is reasonable that a deviation in normal force can be regarded as an indicator of dynamic stall onset. In some cases, however, the vortex is not strong enough that an additional normal force cannot be observed clearly, then an alternative using the maximum normal force can be taken as the indicator of stall-onset (Fig. 2(a)).

\( C_m \) break:

A break in the aerofoil pitching moment usually indicates stall. In dynamic conditions, the break point in the pitching moment may be well delayed if the pitch rate is large enough. This is because of the time required to form a concentrated dynamic vortex that detaches from leading-edge region and traverses over the chord, causing the centre of upper surface pressure moving aft to the trailing edge, and hence inducing a large nose-down pitch moment. It is appropriate that the break point in pitching moment of \( \Delta C_m = 0.05 \) be taken as an indicator of stall-onset (Fig. 2(b)). However, in practice, experimental pitching moments do not always give a clear break point.
$C_c$ maximum:
The chordwise force depends primarily on the leading-edge pressure suction. Under dynamic conditions, the leading-edge pressure suction may build up to a large value, and induce a large chordwise force. However, when the dynamic stall occurs, the concentrated vortex detaches from the leading-edge region, and the leading-edge pressure suction collapses, causing a collapse in chordwise force. Therefore, the maximum chordwise force is a good indicator of stall-onset for many aerofoils (Fig. 2(c)), unless the aerofoil thickness becomes excessive where the maximum chordwise force could be very difficult to identify. A benefit of using maximum chordwise force is that it would not be influenced by some local effects, such as bubbles at leading-edge region.

$C_d$ deviation:
Due to a reduction in chordwise force and an increase in normal force, a rapid increase in section drag may be expected when stall onset is imminent. Hence, a deviation in profile drag could be used as a stall-onset indicator (Fig. 2(d)).

$C_p$ collapse at Leading Edge(LE):
When dynamic stall occurs, the concentrated vortex detaches from leading-edge region, causing a collapse in LE pressure. Therefore, the LE pressure collapse may be a good indicator for stall onset (Fig. 2(e)). However, this criterion could be easily affected by some local disturbances, such as leading-edge bubbles, or the compressible flow at the leading edge region. In these cases, it is not easy to use this criterion to indicate the stall onset.

$C_p$ deviation:
Seto & Galbraith\(^{(10)}\) suggested that early indications of incipient stall may be disguised, or hidden, due to the airloads being calculated by integrating the recorded pressure coefficient values around the surface of the aerofoil. During vortex initiation, the formation of any localised disturbance within the boundary layer might be indicated immediately by the response of the local pressure coefficient. By examining the individual pressure traces, they established a criterion for indicating the onset of stall. Stall onset was assumed to have occurred when the pressure coefficient diverged at somewhere near the $\frac{1}{4}$-chord location (Fig. 2(f)).

Sheng \textit{et al}\(^{(9,11)}\) have compared different stall-onset criteria for the NACA 0012 aerofoil at low Mach numbers, and concluded that all these stall onset criteria gave much similar results for this aerofoil. Obviously, this may not necessarily be the case for other aerofoils or higher Mach numbers. Wilby\(^{(2)}\) compared three stall-onset criteria, namely $C_N$ maximum, $C_m$ break and trailing-edge pressure diverging/leading-edge suction collapsing, for the RAE-series aerofoils and concluded that these three criteria cannot be used to identify stall onset with any precision. Wilby chose the leading-edge suction peak/collapse, which rises with angle-of-attack in attached flow, but collapses when the flow separates (dynamic vortex detaches from the leading-edge region), as the indicator of dynamic stall onset.

3.0 A NEW INDICATION OF STALL ONSET

Sheng \textit{et al}\(^{(9,11)}\) developed a new criterion to identify stall-onset for aerofoils at low Mach numbers. It comprises of a stall-onset angle indication from measured data, a mathematical method to derive the stall-onset parameters directly from 2D aerofoil ramp-up tests, and a method for indicating stall-onset in dynamic conditions. Sheng \textit{et al}\(^{(12)}\) have also extended this method to oscillatory tests of aerofoils.
Figure 3 gives a typical example of stall-onset angles against the reduced pitch rates at low Mach numbers, where two piecewise functions are well representing the stall-onset angles in a large range of reduced pitch rate. Mathematically, Sheng et al used two piecewise functions to represent the stall-onset angle, $\alpha_{ds}$, as:

$$\alpha_{ds} = \begin{cases} 
\alpha_{ss} + \left(\alpha_{ds0} - \alpha_{ss}\right)\frac{r}{r_0} + T_{\alpha} r, & r \leq r_0 \\
\alpha_{ds0} + T_{\alpha} r, & r > r_0 
\end{cases} \ldots (1)$$

where $\alpha_{ss}$ is the static stall angle from a quasi-static test, and $\alpha_{ds0}$ and $T_{\alpha}$ are the stall-onset parameters, derived from 2D aerofoil ramp-up tests.

In the cases of low Mach numbers, the experimental data shows that, when the reduced pitch rate is larger than a certain value, $r_0$, the stall-onset angle increases linearly with the reduced pitch rate. A mathematical expression for the relationship is

$$\alpha_{ds} = \alpha_{s} + \lambda_{\alpha} r \ldots (2)$$

A least square method is employed to fit Equation (2) to the experimental data where the reduced pitch rate is larger than $r_0$ (see Fig. 3). Then the stall-onset parameters can be obtained:

$$\begin{cases} 
\alpha_{ds0} = \alpha_{s} \\
T_{\alpha} = \lambda_{\alpha} 
\end{cases} \ldots (3)$$

This is a general case for stall-onset for a large range of reduced pitch rate.

However, aerofoils may behave differently at higher Mach numbers. Data for RAE 9651 dynamic tests (see Humphreys(13)) became available, covering a higher Mach number range of $M = 0.3 - 0.7$. The reduced pitch rate is relatively low due to higher free stream velocity. In all the testing conditions, the maximum reduced pitch rate is just above 0.01 ($r_0 = 0.01$), occurring at $M = 0.3$ at a very high pitch rate of $\alpha_{s} = 1468.9^\circ/s$. For the RAE 9651 aerofoil unsteady tests, the reduced pitch rates are considered as $r < r_0$, therefore, a linear function can represent the stall-onset angles.
The RAE 9651 aerofoil tests were performed in 1983 using equipment that would now be regarded as giving a low sampling frequency. The number of test runs was limited thus the results may be somewhat sparse when compared to more recent tests. As a result there may be more scatter in the results presented by three experienced researchers than in more recent tests. Figures 4-7 show the stall-onset angle variation with reduction pitch a range of Mach numbers, $M = 0.3$ to 0.6. For each Mach number the results for the clean and gritted aerofoil (forced boundary layer transition) are presented separately.
In these figures, Onset No. 1 and Onset No. 2 are the results obtained by two different researchers. The third researcher presented both the stall-onset angle, $\alpha_1$, and a corresponding angle, $\alpha_m$, via a trailing-edge separation modification. The data is scattered, nevertheless, a linear increase in the stall-onset angle with the reduced pitch rate can still be observed. The present authors decided a linear representation for each case (solid line), from which a linear relation between the stall-onset angle and the reduced pitch rate is given simply by:

$$\alpha_{ds} = \alpha_{ss} + rT_\alpha$$

where $\alpha_{ss}$ and $T_\alpha$ could be derived from the stall-onset angles. Table 1 lists all the derived stall-onset parameters for Mach numbers of 0·3-0·6.

<table>
<thead>
<tr>
<th>Mach</th>
<th>Clean aerofoil $\alpha_{ds0}$</th>
<th>$T_\alpha$</th>
<th>Gritted aerofoil $\alpha_{ds0}$</th>
<th>$T_\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0·3</td>
<td>15·6</td>
<td>7·11</td>
<td>15·0</td>
<td>7·11</td>
</tr>
<tr>
<td>0·4</td>
<td>15·5</td>
<td>4·89</td>
<td>14·9</td>
<td>7·85</td>
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<tr>
<td>0·5</td>
<td>12·5</td>
<td>5·35</td>
<td>13·6</td>
<td>0</td>
</tr>
<tr>
<td>0·6</td>
<td>8·0</td>
<td>0</td>
<td>7·6</td>
<td>0</td>
</tr>
</tbody>
</table>

4.0 THE BEDDOES’ DYNAMIC STALL MODEL AND ITS MODIFICATIONS

As early as the 1970s, Beddoes noticed that there were several non-dimensional time delay constants involved in the dynamic stall process, regardless of the types of aerofoil motions. Based on this observation, Beddoes developed his first generation time-delay dynamic stall model (FGM). After several refinements and improvements by Leishman & Beddoes, the latest version of L-B dynamic stall model was finalised in 1993 by Beddoes, herein known as the third generation dynamic-stall model (TGM). It is comprised of indicial functions for the assessment of attached flow, a Kirchhoff equation for the assessment of trailing-edge separation.
which is lagged in unsteady conditions, an adapted Evans-Mort correlation/shock reversal condition\(^{(15)}\) for stall onset prediction, and a process for vortex formation and convection.

Investigations carried out at Glasgow University have illustrated that the applicability of the L-B model is reduced when used to reconstruct the unsteady airloads at low Mach numbers of about 0·12, including dynamic stall. To remedy the deficiencies of the L-B model for low Mach numbers, a revised dynamic stall model is proposed and presented. The following are the main modifications:

\[(i)\] A new stall-onset criterion developed by Sheng et al\(^{(9,11)}\) has been employed to replace the formulation for the Evans-Mort correlation/shock reversal condition expressed in terms of lagged \(C_N\);

\[(ii)\] A separation location look-up table has replaced the exponential functions of separation location;

\[(iii)\] The modelling of the dynamic vortex formation and convection has also been revised;

\[(iv)\] The formula for chordwise force has been revised;

\[(v)\] A new modelling of the return from the stalled state proposed by Sheng et al\(^{(7)}\) has been included;

This model will be referred to as the Glasgow University Dynamic Stall Model (GDM).

In this section, Beddoes TGM is first described and then followed by the modifications made of Sheng et al\(^{(20)}\).

The L-B dynamic stall model

**Forcing expressions**

As for the L-B dynamic stall model, the airloads for attached flow are assessed via indicial functions. To be more general, the forcing terms are, in contrast to using the form of angle-of-attack and pitch rate, given in terms for lift and pitching moment in both circulatory contributions, \(\eta_n\), \(\eta_m\), and non-circulatory or impulsive contributions, \(\lambda_n\), \(\lambda_m\), (see also Beddoes\(^{(19)}\)). The detailed forcing representations are:

\[
\eta_n(t) = \frac{1}{\pi} \int_{0}^{\pi} (1 + \cos\theta) \frac{w_n(\theta, t)}{V} \, d\theta \quad \ldots (5a)
\]

\[
\eta_m(t) = \frac{2}{\pi} \int_{0}^{\pi} (\cos\theta + \cos2\theta) \frac{w_n(\theta, t)}{V} \, d\theta \quad \ldots (5b)
\]

\[
\Delta\lambda_n(t) = \frac{1}{2} \int_{-1}^{1} \frac{\Delta w_n(x', t)}{V} \, dx' \quad \ldots (5c)
\]

\[
\Delta\lambda_m(t) = \frac{1}{2} \int_{-1}^{1} \frac{\Delta w_n(x', t)}{V} (x' - a) \, dx' \quad \ldots (5d)
\]

with boundary condition,

\[
w_n(x, t) = \frac{\partial z_n(x, t)}{\partial t} + V \frac{\partial z_n(x, t)}{\partial x} \quad \ldots (6)
\]
on the aerofoil surface.
For a pure pitching motion of an aerofoil at its \( c/4 \) axis,

\[ \omega_a = V \alpha + \frac{c}{2} \dot{\alpha} \]  

\( \ldots (7) \)

**Airloads for attached flow**

For a step change in the forcing, the induced changes in forces are obtained as follows.

Circulatory normal force:

\[ \Delta C_n^c(s) = C_{n_0} \Delta \eta_n \left( 1 - A_1 e^{-b_1 s} - A_2 e^{-b_2 s} - A_3 e^{-b_3 s} \right) \]  

\( \ldots (8a) \)

Circulatory moment:

\[ \Delta C_n^l(s) = \Delta \lambda_n \frac{4}{M} e^{-s/T_j} \]  

\( \ldots (8b) \)

Impulsive normal force:

\[ \Delta C_n^i(s) = \Delta \lambda_n \frac{4}{M} e^{-s/T_j} \]  

\( \ldots (8c) \)

Impulsive moment:

\[ \Delta C_m^l(s) = -\Delta \lambda_m \frac{4}{M} e^{-s/T_j} \]  

\( \ldots (8d) \)

The coefficients are given as follows;

\[
\begin{aligned}
A_1 &= 0.165, \\
A_2 &= 0.335, \\
A_3 &= 0.5 \\
b_1 &= 0.05, \\
b_2 &= 0.222, \\
b_3 &= 0.8 / M, \\
T_m &= M/2, \\
T_j &= \frac{c (1 + 3M)}{a} / 4
\end{aligned}
\]  

\( \ldots (9) \)

**Airloads for separated flow**

For aerofoils with trailing-edge separation, the Kirchhoff flow equation is used to obtain the nonlinear normal force. Under dynamic conditions, the boundary layer development is delayed, and it can be represented by a delayed separation function, \( f' \), in case of dynamic condition.

\[ C_n = C_{n_0} (\alpha - \alpha_n) \left( \frac{1 + \sqrt{f'}}{2} \right)^2 \]  

\( \ldots (10a) \)

the chordwise force has been revised to,

\[ C_c = \eta C_{n_0} (\alpha - \alpha_n) \sqrt{f'} \]  

\( \ldots (10b) \)
Given the normal and chordwise forces, the drag may be calculated by,

\[ C_d = C_n \sin \alpha - C_c \cos \alpha + C_{d0} \] \hspace{1cm} (10c)

For the pitching moment, Leishman and Beddoes\(^5\) proposed the formula:

\[ \frac{C_m - C_{m0}}{C_n} = k_0 + k_1 (1 - f') + k_2 \sin (\pi f'') \] \hspace{1cm} (10d)

Beddoes\(^19\) employed a piecewise exponential function to represent the separation location, \( f \), of a static case, in a form of:

\[
\begin{align*}
  f(\alpha) &= 1 - 0.4 \exp \left( \frac{\alpha - \alpha_i}{S_1} \right) \quad \alpha \leq \alpha_i \\
  f(\alpha) &= 0.02 + 0.58 \exp \left( \frac{\alpha_i - \alpha}{S_2} \right) \quad \alpha > \alpha_i
\end{align*}
\] \hspace{1cm} (11)

Under dynamic conditions, the flow separation location is usually delayed due to the dynamic effect.

\[ \Delta f''(s) = \Delta f(s) \left[ 1 - \exp \left( -\frac{s}{T_f} \right) \right] \] \hspace{1cm} (12)

**Airloads due to dynamic vortex**

As it is noted a significant feature of dynamic stall is the linear overshoot in the normal force, compared to the static case. However, at least for low Mach numbers, there is also an additional rapid enhancement of \( C_N \) after stall initiation. This is due to the separated shear layer being retained above the aerofoil surface in the form of a dynamic-stall vortex and its subsequent convection downstream over the aerofoil. Leishman had modelled the general process\(^{17,18}\), but not for the additional overshoot of \( C_N \) at low Mach numbers. Beddoes\(^19\) revised the modelling and proposed a different formula by adding a value to the delayed separation location so to produce an additional normal force for the vortex shedding. This adjustment indeed produces an additional overshoot in normal force when the vortex is strong enough, but it might also produce a non-physical value of delayed separation location e.g. greater than unity.

1) The further overshoot in normal force after dynamic stall-onset, is represented by a further delay in separation location:

\[ \Delta f''(s) = \Delta f'(s) \left( 1 - e^{\frac{s}{T_f}} \right) \] \hspace{1cm} (13)

2) Vortex shedding

\[ V_s = \sin \left( \frac{\pi \tau}{2T_v} \right) \quad \text{for } 0 < \tau < T_v \] \hspace{1cm} (14a)

and

\[ V_s = \cos \left( \frac{\pi (\tau - T_v)}{T_{rz}} \right) \quad \text{for } \tau > \] \hspace{1cm} (14b)
Beddoes\textsuperscript{(19)} used the vortex shape functions to modify the separation locations for normal force and for pitching moment by:

\begin{align}
\frac{d}{dt} f_L &= f'' + [f' - f] \times V_x \quad \ldots (15a) \\
\frac{d}{dt} f_M &= f'' + [f' - f] \times V_x \quad \ldots (15b)
\end{align}

Modifications to the L-B model

\textbf{New stall-onset criterion}

In the modified dynamic stall model, an important change is the replacement of the prediction of stall-onset. In the new stall-onset criterion of Sheng \textit{et al.}\textsuperscript{(9,11)}, the way to determine the stall-onset angle and the method to derive the stall-onset parameters have been introduced. The prediction of stall-onset angle is performed as following. Calculate the lagged angle-of-attack, $\alpha'$, via a time delay constant, $T_\alpha$,

\begin{equation}
\Delta \alpha'(s) = \Delta \alpha(s) \left[ 1 - \exp \left( -\frac{s}{T_\alpha} \right) \right] \quad \ldots (16)
\end{equation}

then stall-onset is said to occur when;

\begin{equation}
\alpha' \geq \alpha_{ss} \quad \ldots (17)
\end{equation}

and the delayed boundary-layer separation, $f'$, can be also calculated from the lagged angle-of-attack, as

\begin{equation}
f'(\alpha) - f(\alpha') \quad \ldots (18)
\end{equation}

\textbf{Separation location look-up table}

As shown by Sheng \textit{et al.}\textsuperscript{(20)}, instead of using the curve fitting postulated by Beddoes, the use of a look-up table for the separation locations ($f$-parameter) is easier to use and tends to give more accurate predictions for aerofoil forces, especially after the stall-onset. Figure 8 shows the measured separate points (o) used in the GDM as compared to Beddoes' original curve-fitting approach (dotted line), for reconstructing the unsteady airloads.
The combination of using the look up table of separation points, together with the improvements described earlier in the GDM, the corresponding normal force and pitching moment reconstructions are shown in Figs 9(a) and 9(b). From these figures, it is clearly seen that much better correlations were obtained against test data than those of using Beddoes’ method.

For the reconstructions, the parameters employed are listed in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Beddoes’ exponential function</th>
<th>Look-up table</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{N_{\infty}}$</td>
<td>0.0983</td>
<td>0.0259</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>0.75</td>
<td>-0.02</td>
</tr>
<tr>
<td>$C_{m_{0}}$</td>
<td>0.03</td>
<td>0.03</td>
</tr>
</tbody>
</table>

### 4.2.3 Airloads due to dynamic vortex

The present authors have revised the understanding of the vortex formation and convection as follows:

- The vortex forms and then detaches from the leading-edge region, inducing an overshoot in normal force as if the flow is still attached to the upper surface. Hence, its effect could be represented by a further boundary-separation location delay,

- At low Mach numbers, an additional overshoot in normal force occurs due to the vortex convecting across the chord. By referring to the suggestion of Beddoes, the additional normal force is believed to be proportional to the difference between the delayed separation location and its corresponding value in the steady case,

- The large nose-down pitching moment is induced due to the vortex convection over the chord causing movement of the pressure centre. Its strength is proportional to the vortex normal force, whilst its position is calculated by a similar procedure proposed by Leishman et al.

Based on the descriptions above, the airloads in vortex formation and convection may be expressed in the same shape functions (Equations (13), (14), the differences are in the computation of the vortex forces. The new DS model makes a further delay in the boundary layer separation, followed by an additional overshoot of normal force due to the dynamic vortex:
the vortex induced pitching moment;

\[ \Delta C_{av} = B_1 \left[ f' - f \right] \times V \]

the coefficients \( B_1 \) and \( B_2 \) depend on individual aerofoils.

4.2.4 Chordwise force

Chordwise force has been revised by Sheng et al.\(^6\), to produce a negative chordwise force when the flow is fully separated.

\[ C = \eta C_{av}(\alpha - \alpha_q) \left( \sqrt{f'} - E_0 \right) \] \quad \ldots (20)

\( E_0 \) is a modification factor which varies between aerofoils and is typically about 0.15.

4.2.5 Return to attached flow from the stalled state

The modelling method for the return from stalled state is due to Sheng et al.\(^7\). A linear fit is applicable for \( \alpha_{min} \) of ramp-down tests (solid triangles in Fig. 10),

\[ \alpha_{min} = \alpha_{min0} + \lambda_2 r \] \quad \ldots (21)

then a least square method is used to obtain \( \alpha_{min0} \) and \( \lambda_2 \) from the measured data, and a constant time delay, \( T_r \), is defined by;

\[ T_r = \lambda_2 \] \quad \ldots (22)

From the above method both the required duration (\( T_r \)) of the convective phase and its implied reattachment onset angle (\( \alpha_{min} \)) may be obtained. As discussed by Sheng et al.\(^7\), the convective part of the return process, from the fully stalled state, starts when \( \alpha_{min0} \) is achieved and the \( C_N \) then follows the average gradient of the appropriate normal force curve (\( dc_n/d\alpha \)) until the appropriate time delay \( T_r \) is expired. After this, an exponential function allows the \( C_N \) value to return to the fully attached flow curve.

![Figure 10. Re-attaching finish angle, \( \alpha_{min} \), of S809 ramp-down tests and its linear fit.](image-url)
5.0 ANÁLISIS Y DISCUSIÓN DE RESULTADOS

In this section, the predictions are conducted only for the ramp-up tests at Mach number of 0.3 for the RAE 9651 aerofoil. For comparison, the results from the Beddoes model are also presented. The parameters used in dynamic stall models are partially given in Tables 1 and 2. Some other parameters employed are given in Table 3, where the values with asterisk are only used by the Beddoes DS model.

<table>
<thead>
<tr>
<th>$C_{Ni}$</th>
<th>$T_p$</th>
<th>$T_f$</th>
<th>$T_v$</th>
<th>$T_{st}$</th>
<th>$B_1$</th>
<th>$B_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5*</td>
<td>1.7*</td>
<td>3.0*</td>
<td>8.0</td>
<td>6.0</td>
<td>0.4</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Generally, the new GDM model gives better predictions than those of Beddoes, both in normal force and pitching moment.

Figure 11. Nominal pitch rate = 200deg/s (RAE 9651, M = 0.3).

Figure 12. Nominal pitch rate = 600deg/s (RAE 9651, M = 0.3).
In the case of low pitch rate of 200°/s, the new model gives very good reconstructions in both normal force and pitching moment (Fig. 11). The very good reconstructions of normal force and pitching moment can be seen in the case of pitch rate of 600°/s (Fig. 12.)

In the case of pitch rate of 800°/s (Fig. 13), the reconstructions from GDM model are still quite good. Since the experiment for this pitch rate has been conducted in much lower Reynolds number than the cases of lower pitch rates. The analytical stall-onset angle from experimental data was even smaller than that of pitch rate of 600°/s in high Reynolds number. This is a quite strange case which remains to be further understood.

In the case of highest pitch rate of 1,600°/s (Fig. 14), the normal force reconstruction agrees with the experimental data very well before and shortly after stall-onset. In the larger angle-of-attack, due to the limitation of practical pitch motion, the pitch rate after stall-onset may reduce. While in the reconstruction, the pitch motion is considered in a constant speed. For the pitching moment, the pitching moment break-point is well captured, but over-predicts the value before stall-onset.

Given the scatter in the assessed results of stall-onset angles and given that the stall-onset parameters have to be derived from it, the reconstructions of normal force and pitching moment in all cases can be considered as very encouraging.

Based on Wilby\cite{2}, the sample number per cycle in the unsteady tests of the RAE 9651 aerofoil were normally 32 or 64. Exploring the time history of the data, it can be seen that only about
half of the time was used to record the ramp-up process of a sweeping angle-of-attack of 32°, in the high pitch rate, only a third of the recording time. If the 32 points per cycle were used, it meant that in the unsteady experiments, the data acquisition system recorded the data for every 2°. If 64 points per cycle were used, the recorded data were for every 1°. This resolution is still not high enough to capture some major dynamic phenomena.

Given the quality and the sparseness of the data, the analysis of the experimental data was a real challenge to the authors. In the analysed results presented in this paper, the authors made much effort to achieve good decisions through the scattered results.

From these results, the stall-onset parameters were derived, and the ramp-up airloads at Mach = 0·3 were reconstructed. From the results presented, it is found that the new model is capable of reconstructing the unsteady airloads very well at the lower Mach number typically seen on helicopter rotor.

6.0 CONCLUSIONS

A new dynamic stall model, based on improvements to the original Beddoes’ model, has been described and demonstrated on the RAE 9651 aerofoil at M = 0·3. The following conclusions are drawn;

1) The reconstructions of normal force and pitching moment for ramp-up tests at Mach = 0·3 are very good when compared to the measured data. From all the comparisons, the new model gives better results than the Beddoes third generation model.

2) The look-up table of separation location works well in reconstructing the unsteady airloads. It presents better results than those from the piecewise exponential functions.

3) The new model captures the stall-onset very well, though the parameters have to be derived from the very scattered data. In the case of high pitch rate, differences can be seen after stall-onset, but the stall-onset angle is still well-predicted by the new model. In the reconstruction of pitching moment, the break point in pitching moment is also well captured, but over-predicts the pitching moment before stall-onset.

4) The quality and sparseness of the experimental data have made the decision of the stall-onset angles somewhat subjective.

7.0 RECOMMENDATIONS

At this point, the application of the new Glasgow University Dynamic Stall model is limited to ramp-up tests at M = 0·3 for the RAE 9651 aerofoil only. However the benefit of this improved model over Beddoes’ third generation model has been demonstrated. Thus the recommendations are that correlation should be extended to oscillatory tests, and to other Mach numbers and other aerofoils. In addition, the method lends itself readily for practical application and it should be implemented into rotor load calculations.

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