Aeroelastic effects of battens on the flight dynamics of a MAV

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ABSTRACT

Wing flexibility is well established as providing gust rejection and delayed onset of stall for micro air vehicles. As such, many designs adopt wings of a flexible material mounted onto a skeleton comprised of a stiff leading-edge spar and stiff chordwise strips, called battens. These battens are shown to provide additional strength at localised regions of the wing and thus improve the gust rejection and delay stall; however, their effect on the flight dynamics is less studied. Using a numerical modeling approach, this paper explores a design space of vehicles with a varied number of wing battens mounted onto a baseline vehicle with a flexible wing. The battens are modelled as stepwise changes in torsional stiffness along the wing span. The resulting trim characteristics, static stability metrics and flight dynamics are evaluated. The battens are shown to improve gust rejection but otherwise have a complicated effect across the design space. A reduction in the number of battens improves the longitudinal static stability derivative slightly but lowers the lateral and directional static stability. The damping is decreased for the short period mode and increased for the phugoid and dutch roll modes as the number of battens is reduced.

Paper No. 4033. Manuscript received 18 July 2013, revised version received 19 November 2013, accepted 27 January 2013.
NOMENCLATURE

b wing span

\( \frac{C_{La}}{C_{L}} \) aircraft lift-curve slope, 1/rad

\( C_{Lb} \) lateral static stability derivative, 1/rad

\( C_{Lc} \) lateral static stability cross derivative, 1/rad

\( C_{Ld} \) aileron control power derivative, 1/rad

\( C_{Le} \) longitudinal static stability derivative, 1/rad

\( C_{Le} \) elevator control power derivative, 1/rad

\( C_{Lq} \) pitch damping derivative, 1/rad

\( C_{Lr} \) directional static stability derivative, 1/rad

\( C_{Ls} \) rudder control power derivative, 1/rad

\( C_{Lr} \) yaw damping derivative, 1/rad

\( EI \) bending stiffness, Nm²

\( GJ \) torsional stiffness, Nm²

\( q \) aircraft pitch rate, deg/s

\( u \) \( x \)-component of aircraft velocity, ms⁻¹

\( V_{\infty} \) aircraft airspeed, ms⁻¹

\( x \) longitudinal dimension, positive upstream, m

\( y \) lateral dimension, positive out right wing, m

\( z \) vertical dimension, positive up, m

\( \alpha \) aircraft angle-of-attack, deg

\( \beta \) aircraft angle-of-sideslip, deg

\( \delta_{a} \) aileron deflection, deg

\( \delta_{e} \) elevator deflection, deg

\( \Gamma \) wing dihedral angle, deg

\( \zeta \) damping ratio

\( \theta \) aircraft pitch angle, deg

\( \tau \) wing twist about the \( y \)-axis, deg

\( \phi \) aircraft bank angle, deg

\( \psi \) aircraft yaw angle, deg

\( \omega_{n} \) natural frequency, Hz

1.0 INTRODUCTION

A batten is a slender piece of relatively stiff material used to strengthen a structure. Such pieces are fairly common in designs for many communities. They are ubiquitous in nautical applications such as constraining the shape of a sail or the outer mold lines of a hull. They are also an important component of airships\(^{(1,2)}\) and spacecraft structures\(^{(3)}\).

The concept of battens is even prevalent in biology\(^{(4)}\). Bats have bones in their wings that provide localised stiffness in a manner similar to battens\(^{(5)}\). Insects wings are made of membranes reinforced with blood-filled veins that provide additional stiffness like a batten\(^{(6)}\). The concept actually existed long ago in pterosaurs, which had actinofibrils that were essentially battens and helped provide stiffness in localised parts of their wings\(^{(7)}\).

Battens are also an integral part of a fixed-wing class of micro air vehicles (MAVs). A distinguishing feature of these vehicles is high flexibility in the wings. Such flexibility has been empirically demonstrated by pilots to ease pilot workload in maintaining level flight due to the
passive washout that occurs in response to loading\(^8\). As such, the wings are constructed using a low-stiffness material within which a high-stiffness skeletal frame, including battens, is embedded. Many aircraft in this class were constructed using battens and successfully flown\(^8,14\).

The role of wing flexibility was extensively analysed using computational fluid dynamics\(^15\). Fluid and structural solvers were coupled together to study the aerodynamic characteristics of a membrane wing in low Reynolds number flow\(^16,17\). A membrane wing with three battens showed increased lift due to passive inflation as well as a lower effective angle-of-attack due to the trailing-edge deflection. The passive inflation also had the effect of delaying stall. Other structural modeling of a batten-reinforced membrane wing was successfully compared to experimental data\(^18\).

The computational effects were validated using wind tunnels. One study demonstrated the pronounced delay in stall that occurs for flexible wings as compared to rigid wings for a MAV\(^14\). Another detailed testing and characterisation of a membrane wing showed improved longitudinal stability and improved gust rejection over a rigid wing\(^19\).

The batten configuration is generally not chosen through a rigorous design methodology; instead, the battens are often chosen using ad hoc metrics or trial-and-error flight testing. Some amount of tailoring was introduced by selecting the battens in order to balance the amount of deformation against the need to maintain a desired wing shape\(^20\). A topology optimisation was used to design a pattern of stiffness elements, which are not restricted to classic batten shapes, to enhance properties such as gust rejection\(^21\).

This paper considers the relationship between batten configurations and flight dynamics. Consideration of these effects may provide the aircraft designer with a new design parameter to alter the open loop behaviour of the aircraft in the conceptual design process. The purpose may be to avoid undesirable effects on the flight dynamics or to take advantage of a certain batten configuration that is beneficial to the mission.

In particular, the effect of the number of battens is demonstrated on the trim characteristics and static stability as well as the frequency and damping of the flight modes. A numerical modeling approach is used to estimate these effects.

### 2.0 BASELINE MODEL

#### 2.1 The GenMAV aircraft

The vehicle used in this study is the Generic MAV (GenMAV), developed at the Munitions Directorate of the Air Force Research Laboratory at Eglin Air Force Base, Florida\(^22,23\). The GenMAV was designed to provide an open, versatile platform for MAV research and development.

![Figure 1. The GenMAV aircraft.](image)
The GenMAV consists entirely of carbon fibre construction with a wingspan of 61cm, a fuselage length of 42cm, horizontal tail width of 30cm, and a vertical tail height of 12cm. It uses a fixed, high-wing configuration with a conventional tail, as illustrated in Fig. 1. The aircraft is a bank-to-turn vehicle using elevons to control pitch and roll and has a flight speed of 15ms⁻¹. The elevon chord length is 50% of the horizontal tail chord length. The flight-ready aircraft has a mass of 1·02kg. The wing of the GenMAV is designed with a 7° dihedral and a 5° incidence angle with slightly varying aerofoils along the span. The aircraft centre of gravity is located on the longitudinal axis at \( x = 0.163 \).

The structural properties of the GenMAV wing are of primary concern. The carbon fibre layup tapers from three plies in the front quarter-chord of the wing to a single ply at the trailing edge, except in the inner span where it tapers to two plies at the trailing edge. Each ply consists of a bidirectional fibre cloth where the fibres are oriented 45° off the aircraft longitudinal and lateral axes. The wing also has three battens of uni-directional carbon fibre aligned in the chord direction at span locations of 0·45\( b \), 0·66\( b \), and 0·85\( b \).

### 2.2 ASWING

The numerical tool used in this work is ASWING\(^{24,25}\). ASWING is an integrated tool for the modeling and simulation of aerodynamics, structural dynamics, and control of a flexible aircraft. Aerodynamic modeling is based on a compressible vortex/source lattice method with wind-aligned trailing-vortices and unsteady aerodynamics. The aircraft structure is modelled using nonlinear Bernoulli-Euler one-dimensional beams, where the relevant structural and aerodynamic properties are specified along the beam.

Instead of coupling the aerodynamics, structural dynamics, flight dynamics, and control dynamics through influence matrices, ASWING combines the governing equations into a single, coupled nonlinear system. The equations are then solved with a Newton method. Linearised frequency-domain computations are accomplished with an eigenmode analysis, allowing for investigations of the flight dynamics, control response, and flutter. The resulting implementation can rapidly obtain solutions and is well-suited for the problem of conceptual aircraft design.

The various components of ASWING have been separately validated against analytical cases. For example, a modal analysis of a free-free beam results in less than one percent error in the beam natural frequencies and shows the method is 2nd-order accurate. The coupled unsteady aerodynamics and structural formulation is validated using a 2D bending/torsion flutter case, resulting in very good agreement between the numerical analysis and the exact solution\(^{25}\).

NASA used ASWING for the mishap investigation of the 2003 Helios crash to analyse and predict flight data\(^{26}\). The primary application was to estimate the spanwise lift distribution in the presence of vertical gusts and to estimate the behaviour of the phugoid mode. ASWING correctly indicated negative phugoid damping in the presence of high wing dihedral. Although ASWING slightly under-predicted the amount of dihedral needed before the negative damping would occur, the mishap report did not consider the differences to be significant because of the uncertainty in the flight test data. Below 9ft of wing tip deflection, ASWING predicted a stable phugoid mode, which agreed with the flight test data.

### 2.3 The GenMAV model

#### 2.3.1 Model development

The ASWING model of the GenMAV is constructed by importing the geometric, mass, and aerodynamic properties of a previously-existing rigid-body model which was developed in AVL,
which is a software package for the design and analysis of aircraft using a rigid-body, vortex lattice approach\footnote{RDL}. In both models, the aerodynamic reference centre is located at the centre of gravity. It is important to note that the GenMAV’s elevons have been separated into ailerons and an elevator for this analysis.

The structural properties of the GenMAV’s wing were obtained using a dynamic stereoscopic digital image correlation system\footnote{Cardinal et al.}. The wing was subjected to loading at various points along the span while the root was clamped. The resulting deflections were used to estimate the bending and torsional stiffness. The wing was found to have a span-averaged bending stiffness of 1.77Nm\(^2\) and torsional stiffness of 0.32Nm\(^2\).

The elastic axis of the wing was determined by applying a load at different chordwise positions at a constant spanwise location to produce varying amounts of wing twist. Linear regression was used to determine the chord location at which the twist was zero. The resulting data were used to estimate the position of the shear centre at each spanwise location. A line connecting the shear centres defines the elastic axis. It was found that the elastic axis lies slightly aft of the quarter-chord line at approximately 28\% chord at all spanwise locations.

### Table 1

<table>
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<th></th>
<th>(C_{La})</th>
<th>(C_{ma})</th>
<th>(C_{mq})</th>
<th>(C_{lb})</th>
<th>(C_{lp})</th>
<th>(C_{nr})</th>
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<td>-12.3</td>
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<td>-14.3</td>
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<td>0.0916</td>
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### Table 2

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<th>Roll (\omega_n) (Hz)</th>
<th>Dutch Roll (\zeta(-)) (Hz)</th>
<th>Phugoid (\omega_n) (Hz)</th>
<th>(\zeta(-)) (Hz)</th>
<th>Short Period (\omega_n) (Hz)</th>
<th>(\zeta(-)) (Hz)</th>
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<td>-26.05</td>
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<td>-31.93</td>
<td>1.40</td>
<td>0.23</td>
<td>0.14</td>
<td>0.10</td>
<td>2.77</td>
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</table>

#### 2.3.2 Flight dynamics validation

The rigid GenMAV model is first compared to the previously existing rigid AVL model\footnote{Badcock & Lind}. The static stability derivatives and pole locations agree well, as seen in Tables 1 and 2. The largest difference in the static stability derivatives is the value of \(C_{La}\), which differs by 29\%. The spiral convergence in the AVL model is a spiral divergence in the rigid ASWING model. Otherwise, the pole locations are within reasonable agreement. These differences between ASWING and AVL could come from changes to the aerodynamic modeling. ASWING uses a lifting line theory approach while AVL uses a full vortex lattice approach.

The flexible GenMAV model is validated using flight test data. Flight equipment on the GenMAV included a GPS and Kestrel 2.2 autopilot with telemetry capability. The autopilot provided roll, pitch, and yaw rate data at 20Hz while the GPS sensor recorded position, altitude, and airspeed at the same rate. All flight maneuvers were flown open loop at 15ms\(^{-1}\) and control inputs were recorded as servo positions using the on board data logger.

The control deflections from the flight test are input to the ASWING model where the wing stiffness is set to match the experimentally–determined stiffness of the GenMAV wing.
The comparison of the first maneuver, a full pitch doublet, is shown in Fig. 2. Note that the trim elevator angle predicted by ASWING differed slightly from that measured in flight. As a result, a small offset is present between the flight data and ASWING data shown in Fig. 2(a). The pitch rate response shows good agreement. The response of ASWING leads the flight test response slightly but the magnitude of the response is similar. However, at 3.75 seconds when reverse control input is applied, ASWING over-predicts the response of the vehicle. This could be related to the difference in $C_{mq}$ shown in Table 1.

The second maneuver, a roll doublet, is shown in Fig. 3. Note that in this sign convention, positive aileron deflection results in a positive rolling moment. The responses vary slightly, which may be due to noise in the flight test data, but generally agree well.

The agreement with the flight test results comes despite the many challenges that arise when flight testing MAVs. For example, it can be very difficult for the MAV pilot to achieve a good trim condition while remotely piloting the aircraft. Even with flight testing in smooth air, a very small wind gust can upset the trim condition and complicate the aircraft response. Maneuvers were performed about the yaw axis, but due to complications such as these the data obtained was not of sufficient quality to perform comparisons to ASWING. In light of challenges like these, the comparisons made here are favorable.

### 2.3.3 Structural Validation

The structural characteristics of the GenMAV are demonstrated to be well suited for modeling by ASWING. The one-dimensional computational approach assumes chord-wise rigidity such that the camber is unchanged. The bending stiffness and torsional stiffness are allowed to vary along the span but the chord remains rigid. Such an approach limits the types of platforms that may be accurately modelled; however, it is a suitable approach for the GenMAV.

A modal analysis indicates the assumption of chord-wise rigidity does not cause significant inaccuracies for the first bending mode and first torsion modes of interest. The analysis utilises data from ground vibration testing of a pair of GenMAV wings. One wing has a stiffness which matches the flight vehicle while the other wing has significantly increased stiffness in the chord direction. In each case, the weight of each wing is the same. This was achieved by using small unconnected weights along the battens in the first wing configuration and using small metal rods.
adhered along the chord in the high-stiffness configuration. The original GenMAV wing weighed 23.8g but these test configurations weighed an extra 4.7g due to the added mass.

The vibration test used a wing mounted horizontally with the entire root fixed as a cantilever. A shaker was attached to the root to provide force excitation. The resulting motion was captured using a scanning laser Doppler vibrometer (29,30) at 207 points across the outer span of the wing. A sine sweep from 2Hz to 200Hz over 8 seconds provided the input while the response was measured at 1,600Hz.

The spectrum is shown in Fig. 4 for the low-frequency range. In this case, a trio of peaks are evident corresponding to the span-wise first bending mode near 17.75Hz, a first torsion mode near 33.1Hz, and a span-wise second-bending mode near 75Hz. The frequencies and damping are nearly identical between the two wing configurations for the first bending and first torsion modes.

The mode shapes for the first bending and first torsion modes, shown in Figs 5(a) and 5(b), are unchanged by the increase in chord-wise stiffness. The mode shapes are indistinguishable for the
first bending mode while the modes shapes for the second-bending mode show only slight differences near the outboard trailing-edge.

A two-dimensional slice along a line of constant span is extracted from the mode shapes in Figs 5(a) and 5(b) to give the aerofoil shapes in Fig. 6. In this case, the slice is taken at \( y = 0.65b \) to highlight the region that showed the largest differences in Figs 5(a) and 5(b). The similarity of the first bending mode shape is clearly shown by this slice, as are the differences in the first torsion mode shape. For the first torsion mode, the true-stiffness wing has a worst-case deflection about 28% higher than the high-stiffness wing when comparing motion relative to the node line. This worst-case deflection is actually localised to the regions of \( 0.65c - 0.75c \) and \( 0.85c - 0.95c \) and near the trailing-edge while over two-thirds of the aerofoil differs by less than 5% due to the change in stiffness.

The approximation of chord-wise rigidity is thus considered acceptable for modeling the first bending and first torsion modes of the GenMAV. The variations from chord-wise rigidity to true chord-wise stiffness result in only slight changes to a small portion of the first torsion mode shape. The effects of these mode shapes on the aeroelastic dynamics are similarly accurate.
The flight dynamics were analysed for a set of configurations within a design space, which was constructed to encompass wings with varied number of battens. Each configuration used the geometry and mass of the GenMAV as a baseline where the empennage and fuselage are modelled as rigid elements while the stiffness of the wing was varied across the design space.

The battens in this design space are strips that run chordwise along the wing. Each strip has a width of 1.27 cm and a length that extends along the entire chord at its spanwise location. These locations are chosen to lie equally spaced along the span as illustrated in Fig. 7 for a configuration with 3 battens on each wing.

The initial wing is modelled as carbon fibre with varying thickness. A thin leading-edge spar is modelled as 4-ply carbon fibre while the remainder of the wing is modelled as 1-ply carbon fibre. Such a wing has an approximate bending stiffness of $EI = 0.5 \text{Nm}^2$ and a torsional stiffness of $GJ = 0.1 \text{Nm}^2$.

Battens are introduced into the wing by altering the stiffness properties. The narrow width of the batten precludes its having a significant effect on the bending stiffness; conversely, the torsional stiffness will be strongly affected by the battens. As such, the torsional stiffness increases to $1.0 \text{Nm}^2$ where the battens lie, which approximately correlates to a 4-ply carbon fibre layup. The variation of torsional stiffness across the semi-span is demonstrated in Fig. 8 for configurations ranging from 0 battens to 10 battens along each wing.

Additionally, a configuration is included in the design space which represents a uniformly-stiff wing without any battens. This ‘solid’ wing uses a constant 4-ply construction of carbon fibre so the torsional stiffness is $1.0 \text{Nm}^2$ throughout.

![Figure 7. GenMAV wing geometry with three battens.](image-url)

![Figure 8. Profile of torsional stiffness across the half-span for each batten configuration.](image-url)
This approach gives a good realisation for the torsional stiffness along the span of a batten-reinforced wing but slightly unrealistic approximation for bending stiffness. In a typical batten-reinforced wing, only a small component along the leading edge would possess a high bending stiffness. In this approximation, the bending stiffness remains constant along the chord.

It is also important to note that these stiffness levels are not low enough to represent an insect-like membrane wing. Insect wings typically have stiffness values below 0·01Nm², much lower than the MAV wing considered here. Rather, the wing modelled here represents a thin, highly flexible wing, perhaps made of carbon fibre, with chordwise batten reinforcements.

The flight condition for each model is restricted to straight and level flight with \( V_\infty = 15\text{ms}^{-1} \) at sea-level altitude on standard-day conditions.

### 4.0 RESULTS AND DISCUSSION

Data is presented for configurations with 0 to 10 battens, as well as for a wing modelled as a single piece of carbon fibre (indicated in plots by \( \infty \)).

#### 4.1 Trim characteristics

The conditions are determined at which each configuration in the design space is trimmed. The associated values for angle-of-attack and elevator deflection are shown in Fig. 9. These values indicate the required angle-of-attack increases while the required elevator deflection decreases as the number of battens is decreased. Specifically, the configuration with 10 battens trims at an angle-of-attack of 13·4º and elevator of –10·7º while the configuration with 0 battens trims at an angle-of-attack of 16·3º and elevator of –13·5º which is a change of 17·8% and –20·7%.

The variations in angle-of-attack and elevator deflection are purely a result of aeroelastic effects since each configuration operates at the same airspeed with identical distributions of mass and nominal geometry. In this case, the aeroelastic effects manifest as wing twist. The averaged twist across the wing varies, as shown in Fig. 10, from –7·3º for a configuration with 10 battens to –11·6º for a configuration with 0 battens with the negative sign indicating leading-edge-down twist. The battens, while being geometrically small, change the overall stiffness enough to cause this significant variation in twist to occur.

![Figure 9. Trim \( \alpha \) and \( \delta_e \).](image1)

![Figure 10. Span-averaged wing twist.](image2)
The angle-of-attack and wing twist are actually quite coupled. The wing essentially has an effective angle-of-attack resulting from the addition of the aircraft angle-of-attack and the wing twist. This effective angle-of-attack, as shown in Fig. 11, is nearly constant for each variation in battens. This near constancy implies the configurations with less stiffness are twisting more and the aircraft, including horizontal tail, is increasing the angle-of-attack to compensate. The resulting orientation has the wing generating the same amount of lift for any of the configurations and, thus, the elevator decreasing to reduce the moment it provides as the angle-of-attack is increased as in Fig. 11.

The values for angle-of-attack, elevator deflection and twist in Figs 9 and 10 monotonically trend across the configurations towards the solid-wing configuration, denoted by $\infty$, but do not necessarily converge. Consider that 25 battens would entirely cover the wing but the battens would be connected by a membrane as opposed to being a single, unified piece of carbon fibre. The stiffness is thus quite different for a carbon-fibre wing made of 25 thin pieces or 1 large piece.
4.2 Sensitivity of lift and drag

The effect of battens on gust sensitivity can be somewhat indicated by the lift-curve slope known as $C_{L\alpha}$ and given in Fig. 12(a). This parameter indicates the increase in lift that results from an increase in angle-of-attack which may occur from, among several possibilities, a wind gust. The configurations with fewer battens clearly have a lower lift-curve slope with $C_{L\alpha} = 4.83$ when the wing has 10 battens as compared to $C_{L\alpha} = 4.27$ when the wing has 0 battens. This decrease in the lift-curve slope correlates with the trim characteristics in that the lower stiffness associated with fewer battens causes the wing to deform more when it is loaded and thus generate less lift. Such a significant decrease in lift-curve slope and gust sensitivity agrees with pilot observations.

This variation of lift-curve slope across the design space directly correlates with stiffness. The configurations with less stiffness will deform in response to the increased loading that results from an increase in angle-of-attack. This behaviour is associated with less stiffness due to fewer battens, as shown in Fig. 12(a), or less stiffness due to thinner battens\(^{(20)}\).

The variation of drag with angle-of-attack, known as $C_{D\alpha}$, is given in Fig. 12(b) for the design space. This variation is somewhat counterintuitive $C_{L\alpha}$ in that decreases but $C_{D\alpha}$ increases as the number of battens is reduced. The decrease in lift is accompanied by a decrease in induced drag; however, the increased deformation causes a significant increase in parasitic drag. The resulting drag is actually more sensitive to changes in angle-of-attack as the number of battens is reduced.

4.3 Static stability

Each configuration in the design space is statically stable about the longitudinal axis as shown by $C_{m\alpha}$ in Fig. 13. This metric relating static stability is more statically stable longitudinally for the configurations with a reduced number of battens.

The improvement in static stability is approximated by noting the contributions of lift and drag from the wing. In this case, Equation (1) derives the resulting pitch moment using the lift and drag as decomposed into their body-axis forces. The lift and drag are further decomposed into first-order functions of the angle-of-attack in Equation (2). This angle-of-attack is small so the approximations in Equation (3) may be used.

\[
M = -z \left( L \cos(\alpha) + D \sin(\alpha) \right) + z \left( D \cos(\alpha) - L \sin(\alpha) \right)
\] \hspace{1cm} (1)
The expression for $C_{m\alpha}$ in Equation (4) results by taking the derivative of Equation (3) and introducing a non-dimensionalising divisor. This approximation estimates that $C_{m\alpha}$ should decrease by 0.08 as the battens decrease from 10 to 0 which is close to the actual decrease of 0.05 computed by the numerical modeling.

$$C_{m\alpha} = -\frac{x}{c} \left( C_{i\alpha} + C_{D\alpha} + 2C_{D\alpha} \alpha \right) + \frac{z}{c} \left( C_{D\alpha} - C_{i\alpha} - 2C_{i\alpha} \alpha \right)$$

(4)

Static stability about the directional axis is appreciably affected by the number of battens. The value of $C_{n\beta}$, which must be positive for the aircraft to be statically stable, decreases from 0.072 to 0.061 as the number of battens decreases from 10 to 0 as shown in Fig. 14.

The variation in $C_{n\beta}$ in Fig. 14 is attributed to aeroelastic effects on lift and drag due to the battens. The dominant contributor to this term is the vertical tail which is unchanged throughout the design space; however, the geometric dihedral generates a small contribution which does indeed vary with number of battens. Consider the yaw moment, $N_y$, in Equation (5) which results from decomposing the lift and drag into body-axis forces and noting the contributions from the right and left wings with a moment arm of $y$. Expanding these forces as affine functions of angle-of-attack and noting the sensitivities are identical for the right and left wings results in Equation (6). This angle-of-attack is small so the approximations in Equation (7) are thus valid. The effect of geometric dihedral is to differ the angle-of-attack on each wing as introduced in Equation (8).

$$N_{\text{dihedral}} = y \left( D_{\text{right}} \cos(\alpha_{\text{right}}) - L_{\text{right}} \sin(\alpha_{\text{right}}) - D_{\text{left}} \cos(\alpha_{\text{left}}) + L_{\text{left}} \sin(\alpha_{\text{left}}) \right)$$

$$= y \left( (D_{\alpha} + D_{\alpha} \alpha_{\text{right}}) \cos(\alpha_{\text{right}}) - (L_{\alpha} + L_{\alpha} \alpha_{\text{right}}) \sin(\alpha_{\text{right}}) \right)$$

(5)
The contribution of the geometric dihedral to static stability is given in Equation (9). This contribution directly results by taking the derivative of Equation (8) and converting into a non-dimensional term. It must be noted that the actual value of geometric dihedral is 12° which results from the nominal value of 7° plus another 5° of bending at trim. Using an effective moment arm of 10cm from the centreline to the centre of pressure, the numerical difference between configurations with 10 battens and 0 battens is thus 0.008 which compares quite reasonably with the difference of 0.011 indicated by Fig. 14.

Note that geometric dihedral is actually a negative contribution to static stability. This contribution is often positive for many aircraft; however, the contribution is negative in this case because the lift component in the body-axis longitudinal direction is large enough to have an effect on the yaw moment. So, the aircraft has a large positive contribution from the vertical tail, which is constant for all configurations, but an increasingly negative contribution from the geometric dihedral as the number of battens is decreased.

The value of $C_{\mu\text{dihedral}}$, which relates static stability in the lateral axis, is given in Fig. 15 for each configuration. These values remain negative to indicate that every configuration is statically stable although the aircraft becomes less stable laterally as the number of battens is reduced. The specific values are $-0.113$ for the configuration with 10 battens and $-0.083$ for the configuration with 0 battens.

The variation observed in Fig. 15 can also be attributed to aeroelastic effects of the battens coupled with the geometric dihedral. Consider the roll moment in Equation (10) which notes the
contributions from each wing. Expanding these forces as affine functions of angle-of-attack and noting the sensitivities are identical for the right and left wings results in Equation (11). This angle-of-attack is small so the approximations in Equation (7) are thus valid. The effect of geometric dihedral is to differ the angle-of-attack on each wing as introduced in Equation (13).

\[
L_{\text{dihedral}} = y(D_{\alpha} \alpha_{\text{left}} \sin(\alpha_{\text{left}}) + D_{\alpha} \alpha_{\text{right}} \cos(\alpha_{\text{left}}) - D_{\alpha} \alpha_{\text{right}} \sin(\alpha_{\text{right}}) - L_{\text{right}} \sin(\alpha_{\text{right}})) \\
= y\left((D_{\alpha} + D_{\alpha} \alpha_{\text{left}}) \sin(\alpha_{\text{left}}) + (L_{\alpha} + L_{\alpha} \alpha_{\text{left}}) \cos(\alpha_{\text{left}})\right) - \left((D_{\alpha} + D_{\alpha} \alpha_{\text{right}}) \sin(\alpha_{\text{right}}) - (L_{\alpha} + L_{\alpha} \alpha_{\text{right}}) \cos(\alpha_{\text{right}})\right) \\
= y\left(D_{\alpha} \left(\alpha_{\text{left}}^2 - \alpha_{\text{right}}^2\right) + (D_{\alpha} + L_{\alpha}) \left(\alpha_{\text{left}} - \alpha_{\text{right}}\right)\right) \\
C_{\beta_{\text{dihedral}}} = -\frac{2y}{b} \Gamma \left(C_{t_{\alpha}} + C_{D_t} + 2C_{D_\alpha} \alpha\right) \\
\]  

The contribution to static stability results by taking a derivative of Equation (13) with respect to angle of sideslip and then converting into non-dimensional representation. This resulting contribution is given in Equation (14). This approximation computes that \( C_{\beta} \) should be \( \cdot034 \) greater for 0 battens than for 10 battens which agrees well with Fig. 15 that notes 0·03 is the difference computed numerically.

\[
C_{\beta_{\text{dihedral}}} = -\frac{2y}{b} \Gamma \left(C_{t_{\alpha}} + C_{D_t} + 2C_{D_\alpha} \alpha\right) \\
\]  

4.4 Control effectiveness

The effect of battens on the control power of the aircraft is negligible, as shown in Fig. 16. The primary control derivatives \( C_{\delta_{\alpha}} \), \( C_{\delta_{\text{left}}} \) and \( C_{\delta_{\text{right}}} \) are essentially unaffected by the changing number

![Elevator control power](image1.png)  
![Aileron and rudder control power](image2.png)

Figure 16. Primary control derivatives.
of battens. Recall that the GenMAV’s elevons have been separated into ailerons and an elevator for this analysis. Accordingly, the ailerons reside on the horizontal tail and their effectiveness is not changed by the battens.

### 4.5 Longitudinal flight dynamics

The phugoid mode shows minor changes in frequency and damping across the design space as shown in Fig. 17(a). The frequency decreases slightly from 0·114 in the 10 batten case to 0·110Hz in the 0 batten case, a –3·7% change. The damping ratio increases from 0·092 in the 10 batten case to 0·099 in the 0 batten case, a 6·8% increase.

The mode shape of the phugoid mode is given by the eigenvector in $t_{b,ph}$ and shows somewhat more variation than either the natural frequency or damping. In particular, the amount of variation in the angle-of-attack is noticeably different between the configurations. The solid wing shows

![Phugoid mode](image)

![Short period mode](image)

Figure 17. Natural frequency and damping of the longitudinal modes.

**Table 3**

| battens per wing: | zero | | ten | | solid |
|-------------------|------|------------------|------|------------------|------|------------------|
| $u$ (ms⁻¹) | 0·237 | 90·225 | 0·232 | 90·994 | 0·201 | 92·929 |
| $q$ (deg/s) | 0·690 | 95·694 | 0·717 | 95·305 | 0·841 | 95·248 |
| $\alpha$ (deg) | 0·263 | 89·519 | 0·197 | 90·116 | 0·085 | 90·648 |
| $\theta$ (deg) | 1·000 | 0·000 | 1·000 | 0·000 | 1·000 | 0·000 |
| $z_{y=-0·3}$ (mm) | 0·345 | –90·745 | 0·435 | –89·925 | 0·595 | –88·150 |
| $z_{y=-0·15}$ (mm) | 0·299 | –90·564 | 0·270 | –89·802 | 0·247 | –88·122 |
| $z_{y=0·0}$ (mm) | 0·000 | –29·396 | 0·000 | 0·000 | 0·000 | 0·000 |
| $z_{y=0·15}$ (mm) | 0·299 | –90·564 | 0·270 | –89·804 | 0·247 | –88·123 |
| $z_{y=0·3}$ (mm) | 0·345 | –90·748 | 0·434 | –89·927 | 0·595 | –88·151 |
| $\tau_{y=-0·3}$ (deg) | 0·590 | –90·541 | 0·649 | –89·759 | 0·168 | –88·022 |
| $\tau_{y=-0·15}$ (deg) | 0·197 | –90·472 | 0·142 | –89·699 | 0·032 | –87·981 |
| $\tau_{y=-0·0}$ (deg) | 0·000 | –99·183 | 0·000 | –89·650 | 0·000 | –86·334 |
| $\tau_{y=0·15}$ (deg) | 0·197 | –90·477 | 0·142 | –89·702 | 0·032 | –87·982 |
| $\tau_{y=0·3}$ (deg) | 0·590 | –90·546 | 0·649 | –89·762 | 0·168 | –88·023 |
a classic behaviour with the angle-of-attack being less than 9% of the pitch angle; however, the ratio increases such that the configuration with 10 battens has an angle-of-attack being nearly 20% of the pitch angle and the configuration with 0 battens has it over 26% of the pitch angle. This non-traditional mode shape results from the wing deforming due to aeroelastic effects. The deformation, as indicated by the eigenvector, is negligible in bending but has a twist at the wingtips that is 60% of the pitch angle.

The battens have the opposite effect on the short period mode as the phugoid mode and act to increase the natural frequency very slightly and decrease the damping as the number of battens is reduced, as shown in Fig. 17(b). The natural frequency increases from 3·37Hz to 3·39Hz as the battens decrease from ten to zero while the damping ratio decreases from 0·27Hz to 0·25Hz, a 8·0% decrease.

The normalised eigenvector components of the short period mode for various batten configurations are listed in Table 4. The mode shape for all configurations retains the classic behaviour of being predominately pitch rate and angle-of-attack with each being nearly 90º out of phase. The aeroelastic effects result in a twist angle that is roughly twice the size of the angle-of-attack or the pitch angle while the bending remains negligible.

The effect of battens on the dynamic response might be approximated using the changing stability derivatives and the classic modal approximations. However, caution is needed because the modal approximations are derived using rigid-body assumptions with no aeroelastic states taken into account. In the present aeroelastic case, the structure is deforming and the moments of inertia are changing. This aeroelastic effect introduces secondary terms to the modal approximations that are beyond the scope of this work.

### Table 4

<table>
<thead>
<tr>
<th>Battens per Wing:</th>
<th>Zero</th>
<th>Ten</th>
<th>Solid</th>
</tr>
</thead>
<tbody>
<tr>
<td>u (ms⁻¹)</td>
<td>0·074</td>
<td>0·060</td>
<td>0·037</td>
</tr>
<tr>
<td>q (deg/s)</td>
<td>20·809</td>
<td>20·326</td>
<td>18·952</td>
</tr>
<tr>
<td>α (deg)</td>
<td>1·000</td>
<td>1·000</td>
<td>1·000</td>
</tr>
<tr>
<td>θ (deg)</td>
<td>0·976</td>
<td>0·958</td>
<td>0·925</td>
</tr>
<tr>
<td>z(0 = 0·3) (mm)</td>
<td>1·546</td>
<td>1·867</td>
<td>2·601</td>
</tr>
<tr>
<td>z(0 = 0) (mm)</td>
<td>0·000</td>
<td>0·000</td>
<td>0·000</td>
</tr>
<tr>
<td>τ(0 = 0·3) (deg)</td>
<td>1·966</td>
<td>2·215</td>
<td>0·653</td>
</tr>
<tr>
<td>τ(0 = 0) (deg)</td>
<td>0·000</td>
<td>0·000</td>
<td>0·000</td>
</tr>
<tr>
<td>τ(0 = 0·3) (deg)</td>
<td>1·967</td>
<td>2·215</td>
<td>0·653</td>
</tr>
</tbody>
</table>

### 4.6 Lateral-directional flight dynamics

The modal parameters of the dutch roll mode, shown in Fig. 18, show some changes in response to decreasing battens. The natural frequency decreases from 1·40Hz to 1·37Hz while the damping ratio increases by 8·5% from 0·31 to 0·34 as the configuration changes from 10 battens to 0 battens.
The mode shape of the dutch roll mode, shown in Table 5, changes across the configurations from a traditional behaviour to a novel behaviour. In particular, the phasing relationships vary noticeably between the angle of sideslip and the roll angle and the yaw angle. The angle of sideslip is 169° out of phase with the yaw angle for the solid wing but the introduction of battens reduces the value such that they vary by only 152° for the configuration with 0 battens. Similarly, the angle of sideslip is less out of phase with the roll angle for the configuration with 0 battens as compared to any other configurations. The magnitude of the roll angle in the motion is also more pronounced as the number of battens is reduced. The phase difference between the wing tip

![Graph showing time constant of the spiral convergence.](image)

**Table 5**

<table>
<thead>
<tr>
<th>battens per wing:</th>
<th>zero</th>
<th></th>
<th>ten</th>
<th></th>
<th>solid</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mag</td>
<td>phase</td>
<td>mag</td>
<td>phase</td>
<td>mag</td>
<td>phase</td>
</tr>
<tr>
<td>$P$ (deg/s)</td>
<td>8.611</td>
<td>105.972</td>
<td>8.773</td>
<td>105.056</td>
<td>9.094</td>
<td>103.196</td>
</tr>
<tr>
<td>$r$ (deg/s)</td>
<td>3.191</td>
<td>-162.726</td>
<td>3.461</td>
<td>-163.822</td>
<td>3.877</td>
<td>-164.243</td>
</tr>
<tr>
<td>$\beta$ (deg)</td>
<td>0.504</td>
<td>-64.836</td>
<td>0.507</td>
<td>-70.601</td>
<td>0.519</td>
<td>-78.180</td>
</tr>
<tr>
<td>$\phi$ (deg)</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$\psi$ (deg)</td>
<td>0.376</td>
<td>87.449</td>
<td>0.398</td>
<td>88.113</td>
<td>0.427</td>
<td>91.430</td>
</tr>
<tr>
<td>$z_{y=0.3}$ (mm)</td>
<td>0.019</td>
<td>72.508</td>
<td>0.019</td>
<td>66.700</td>
<td>0.013</td>
<td>46.524</td>
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<tr>
<td>$z_{y=0.15}$ (mm)</td>
<td>0.014</td>
<td>65.936</td>
<td>0.009</td>
<td>59.242</td>
<td>0.005</td>
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<td>$z_{y=0}$ (mm)</td>
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<td>-103.831</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$z_{y=0.15}$ (deg)</td>
<td>0.028</td>
<td>70.553</td>
<td>0.022</td>
<td>63.032</td>
<td>0.003</td>
<td>47.285</td>
</tr>
<tr>
<td>$\tau_{y=0.3}$ (deg)</td>
<td>0.008</td>
<td>64.351</td>
<td>0.004</td>
<td>55.147</td>
<td>0.001</td>
<td>40.748</td>
</tr>
<tr>
<td>$\tau_{y=0.15}$ (deg)</td>
<td>0.000</td>
<td>-119.058</td>
<td>0.000</td>
<td>-114.434</td>
<td>0.000</td>
<td>-340</td>
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<td>$\tau_{y=0}$ (deg)</td>
<td>0.009</td>
<td>-113.403</td>
<td>0.005</td>
<td>-118.188</td>
<td>0.001</td>
<td>-120.695</td>
</tr>
<tr>
<td>$\tau_{y=0.15}$ (deg)</td>
<td>0.030</td>
<td>-108.857</td>
<td>0.026</td>
<td>-113.822</td>
<td>0.005</td>
<td>-119.328</td>
</tr>
</tbody>
</table>

The mode shape of the dutch roll mode, shown in Table 5, changes across the configurations from a traditional behaviour to a novel behaviour. In particular, the phasing relationships vary noticeably between the angle of sideslip and the roll angle and the yaw angle. The angle of sideslip is 169° out of phase with the yaw angle for the solid wing but the introduction of battens reduces the value such that they vary by only 152° for the configuration with 0 battens. Similarly, the angle of sideslip is less out of phase with the roll angle for the configuration with 0 battens as compared to any other configurations. The magnitude of the roll angle in the motion is also more pronounced as the number of battens is reduced. The phase difference between the wing tip
deflection and the roll angle decreases slightly from being 133° out of phase to 107° out of phase. The phase difference between the wing twist and the roll angle increases slightly. On each wing tip, the deflection and twist remain about 180° out of phase from each other.

The time constant of the spiral convergence decreases from 9s to 6·7s as the number of battens is reduced from 10 to 0 battens, as shown in Fig. 19. The normalised eigenvector components are listed in Table 6:spiral. In the solid wing configuration, the roll component is small, contributing only 0·08° per degree of yaw angle. As the wing becomes more flexible (reducing battens), the contribution from roll grows to 0·25° per degree of yaw angle. At the same time, the yaw rate increases

<table>
<thead>
<tr>
<th>Battens per wing:</th>
<th>zero</th>
<th>ten</th>
<th>solid</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$ (deg/s)</td>
<td>0·063</td>
<td>0·034</td>
<td>0·006</td>
</tr>
<tr>
<td>$r$ (deg/s)</td>
<td>−0·146</td>
<td>−0·110</td>
<td>−0·049</td>
</tr>
<tr>
<td>$\beta$ (deg)</td>
<td>−0·007</td>
<td>−0·005</td>
<td>−0·002</td>
</tr>
<tr>
<td>$\phi$ (deg)</td>
<td>−0·246</td>
<td>−0·178</td>
<td>−0·075</td>
</tr>
<tr>
<td>$\psi$ (deg)</td>
<td>1·000</td>
<td>1·000</td>
<td>1·000</td>
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<td>$z_{y=0·3}$ (mm)</td>
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<td>$\tau_{y=0·15}$ (deg)</td>
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</tr>
<tr>
<td>$\tau_{y=0·3}$ (deg)</td>
<td>−0·001</td>
<td>0·000</td>
<td>0·000</td>
</tr>
</tbody>
</table>

Figure 19. Time constant of the spiral convergence.
in magnitude, indicating a faster recovery from the perturbation. There is no deformation of the structure in the shape of the spiral convergence.

The time constant of the roll convergence, shown in Fig. 20, increases slightly from 0·033s in the 10 batten case to 0·039s in the 0 batten case. The associated mode shapes are given in tab:roll and indicate the behaviour transitions away from a classic pure-roll motion as the number of battens is reduced. The solid-wing configuration has an angle of sideslip that is only 8% of the roll angle whereas the 0 batten configuration has an angle of sideslip that is over 17% of the roll angle. A small amount of wing deflection and twist remain in the response but it is not significant for any of the configurations.
4.7 Structural dynamics

The first-bending mode’s natural frequency is affected very little by the battens, as shown in Fig. 21(a). The damping ratio experiences a stronger effect than natural frequency, however, increasing from 0.36 to 0.41 from 10 to 0 battens, a 13.0% change. The effect on damping may come through aerodynamic coupling of bending and torsion.

The first torsion mode natural frequency sees a slight effect from the battens, decreasing from 35.2Hz to 30.4Hz, a 13.6% decrease. The trend for the damping ratio of the first torsion mode shows no appreciable change from 10 to 0 battens, as shown in Fig. 21(b).

Figure 21. Natural frequency and damping of the structural modes.

Figure 22. Twist of batten-reinforced wings compared to a wing with uniform torsional stiffness.
4.8 Interpretation

One effect of changing the number of battens is to change the average torsional stiffness of the wing, from 0.48 in the case of 10 battens to 0.1 in the case of 0 battens. As the average torsional stiffness decreases, the average wing twist naturally increases in magnitude. This can be seen by comparing the twist of the 10 batten wing to the one-batten wing, shown in Fig. 22. Many of the observed effects follow trends that would normally arise from decreasing torsional stiffness of a solid wing. However, the magnitude of those effects do not correspond to data obtained with the same average torsional stiffness applied as a uniform distribution in a solid wing.

For example, the 10 batten configuration has a uniform bending stiffness of 0.5, an average torsional stiffness of 0.48 and trims at an angle-of-attack of 13.4° and elevator deflection of −10.7°. For a solid wing with a uniform $EI = 0.5$ $GJ$ and $= 0.48$ stiffness distribution, the trim angle-of-attack is 9.3° and the elevator deflection is −6.8°—a difference of −30.6% and 36.5%, respectively. This disparity is illustrated in Fig. 22, where the twist of the 10 batten wing can be compared to the twist of the $GJ = 0.48$ wing. The batten-reinforced wing results in more twist despite the fact that it has the same average stiffness. This is because the flexible portions of the span between the battens are able to twist more significantly than the wing with the uniform distribution. Because of this, the behaviour of an aircraft with a batten-wing cannot be assumed to follow the behaviour of an aircraft with a solid wing of the same average torsional stiffness.

5.0 Conclusion

Battens are shown to have a strong effect on the flight performance of micro air vehicles. The results agree with previous studies that indicate flexibility enhances gust rejection; however, the aeroelastic effects are shown to have a complicated role when considering the flight performance such that some metrics are improved while others are degraded.

Reducing the number of battens improves gust rejection and longitudinal static stability; conversely, this reduction also requires larger angles for trim and degrades static stability in the lateral and directional axes. The effects are equally diverse when considering flight dynamics in that the dutch-roll mode has improved damping while the short-period mode has degraded damping. Furthermore, the nature of the modes change as the batten configuration changes and the aircraft flies noticeably different. These characteristics are especially critical to note when designing an autonomous system that requires both an aircraft and associated autopilot that must compensate the flight modes to maneuver. As such, battens should be considered as another degree of freedom for aircraft design that can be introduced to tailor certain metrics and balance the effects of other design parameters.

Future work should consider the aspect ratio of the wing and how it interacts with the batten effects. Varying batten width could also be considered. In addition, it would be interesting to derive aeroelastic modal approximations which could be used to capture the dynamic effects.

Acknowledgements

The authors are grateful to the United States Air Force Academy, Department of Aeronautics for supporting this work. The numerical analysis was made possible through the use of ASWING. Thanks are due to Professor Mark Drela for the software and his advice.
REFERENCES


