Babinsky’s Demonstration: The Theory of Flight and Its Historical Background

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In memory of
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Alexander Henry Fraser-Mitchell, CEng, FRAeS, MIMechE (1929 - 2014)
who both promoted the science, practice and history of aerodynamics in Britain.

Abstract

In 2012 Holger Babinsky, Professor of Aerodynamics at Cambridge University Engineering Department, provided a wind tunnel demonstration showing that a widely believed explanation for the lift force on a wing is flawed. One purpose of this paper is to provide the correct explanation for this phenomenon. With due regard to the aim of this journal, a further purpose is to outline the historical steps taken in discovering the key elements of that explanation and to draw attention to Britain’s rather late acceptance of it.

1. The Popular Explanation of Flight

An article in the Daily Telegraph of 2012 by Millward and Collins (1) reports Professor Babinsky’s objection to a widely believed explanation for how aeroplanes fly. The easily available on-line version of that article contains a clip from Babinsky’s video demonstration of the flow about a lifting aerofoil which clearly illustrates the flaw in that explanation. Let us begin by outlining the argument to which Babinsky rightly objects.

Figure 1 shows the airflow around a lifting aerofoil. Imagine that two adjacent air elements, marked A and B as shown, are about to reach the aerofoil’s nose. Element A passes above the aerofoil whilst element B passes beneath. Eventually element A reaches the aerofoil’s tail or trailing edge and element B, according to this argument, must reach the tail at the same time so as to regain its position adjacent to element A as shown in Figure 1. Consequently element A, having further to travel over the curved upper surface of the aerofoil, has moved faster than element B. However, an increase in the air’s speed, by the Bernoulli principle, entails a drop in pressure. An appeal to that principle then tells us that, because of its higher speed, element A has moved through a region of lower pressure than that encountered by element B. Thus the aerofoil is sucked upward by this pressure difference between the upper and lower surfaces.

The error in this argument occurs at the point at which it is insisted that elements A and B must regain their adjacent positions at the trailing edge. As Babinsky is quoted (1) as

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correctly saying, there is no physical principle which requires this to happen. Indeed, as Babinsky’s flow patterns revealed by smoke filaments show, element A emerges from the trailing edge ahead of element B, as in Figure 2. Babinsky is not the first to demonstrate the error in this widely believed explanation; over the years, photographs and films have consistently shown the same behaviour. Yet this flawed explanation of flight has survived and still enjoys wide belief, probably because it has the appeal of simplicity. Its origin now seems to be lost in the mists of time although it may be that its use in aircrew training manuals gained for it a popularity which extended to school textbooks and even the occasional British A Level examination paper. It may also be the case that this explanation emerged in Britain before the correct explanation had been accepted here; Britain was rather late in the game in this respect, an aspect which will be examined in the closing section of this article.

![Figure 1](image1.png) Aerofoil flow according to the simple argument.

![Figure 2](image2.png) Aerofoil flow actually observed.

Apart from the insistence on adjacency, however, the remainder of the above argument is correct - so far as it goes. It is true that the airflow speed on the upper surface of a lifting aerofoil is greater than that on the lower surface and that, by the Bernoulli principle, the pressure on the upper surface is less than that on the lower surface. Yet this begs the question: how does an aerofoil induce this airflow behaviour? To answer that question, we must go considerably deeper into the phenomenon. Regrettably, the resulting explanation, whilst avoiding the pitfall of adjacency, will be found to be far from simple. The historical development of the various concepts involved in the explanation, and further details of them, are given in appendices.
2. The Correct Theory of Flight

Rather more than eighty years ago, two experimental investigations in Britain verified a concept which provides the basis for the correct understanding of flight. We will shortly come to that concept, which was relatively new to Britain’s aeronautical research community at the time. Both investigations involved extensive measurements of airflow velocities and pressures around lifting aerofoils in wind tunnels. The first investigation by Bryant and Williams (2) used an aerofoil of a thick and arched or cambered shape, the results appearing in 1924. A similar experimental investigation was undertaken by Tanner (3) as part of a wider study, his results appearing in 1930. His data will be used here, since they were later presented in a form which better serves our purpose. This presentation appeared in 1938 in Reference 4.

Tanner (3) used a symmetrical aerofoil set at 7° incidence to a uniform stream; Figure 3 shows his results replotted (4) to illustrate the streamline patterns. The aerofoil is an RAF 30 section, RAF indicating that its shape had originated at the Royal Aircraft Factory, Farnborough, which in 1918 was renamed the Royal Aircraft Establishment. The solid lines are the theoretically obtained streamlines for this case whereas the dashed lines are those obtained by experiment. The two sets of streamline patterns are seen to be in close agreement. Indeed, the theoretical and observed pressure distributions, shown in Figure 4, are in almost precise agreement, the exception being a small region near the trailing edge; the reason for this will emerge later.

Figure 3  Aerofoil streamline patterns (Tanner (3), 1930) [From Goldstein (4) © By permission of Oxford University Press OUP Material: MODERN DEVELOPMENTS IN FLUID DYNAMICS VOLUME 2 by Sydney Goldstein (1938) Figure 180 from p. 457. www.oup.com]
The first thing to notice in Figure 3 is that it confirms the belief that the airflow increases in speed over the aerofoil’s forward upper surface, the reverse being true on the forward lower surface. By the continuity principle (see Appendix 1), the contraction in the gap between the streamlines over the upper surface and the widening gap on the lower surface show that these changes in flow velocity are taking place. The Bernoulli principle (see Appendix 2) then reveals a lower pressure on the forward upper surface and an increased pressure on the forward lower surface. Indeed, the pressure results (Figure 4) show a strong suction peak on the forward upper surface much larger than the under surface’s slight overpressure. Thus the aerofoil experiences a lifting force which is mainly due to upper surface suction.

![Aerofoil pressure distributions](https://www.oup.com)

Figure 4  Aerofoil pressure distributions (Tanner (3), 1930) [From Goldstein (4) ©
By permission of Oxford University Press OUP Material: MODERN DEVELOPMENTS IN FLUID DYNAMICS VOLUME 2 by Sydney Goldstein (1938) Figure 179 from p. 455. www.oup.com]
A second point to note is that, somehow, the aerofoil has induced the airflow to rise up toward its nose and then to move slightly downward aft of the trailing edge. The momentum of the upflow near the nose is removed continuously by the aerofoil, which then continuously imparts a momentum downflow at the tail. Both rates of change in flow momentum in the vertical plane require the aerofoil to have pushed downward on the airflow. The inevitable reaction on the aerofoil is its upward lift force.

Both views, the continuity/Bernoulli view and the upflow / downflow view, lead to the same conclusion: the aerofoil is experiencing lift. But how does an aerofoil continuously manage to induce this lifting upflow and then downflow?

Before answering that question, let us return to the reason for the experimental investigations conducted by Bryant and Williams\(^{(2)}\) and Tanner\(^{(3)}\). Their purpose was to verify a theoretical result, by then at least twenty years’ old, produced independently by two applied mathematicians, Martin Kutta and Nicolai Zhukovskii (see Appendix 3). Their result, known as the Kutta-Zhukovskii theorem, states that the lift per unit span, \(L\), experienced by an object exposed to a uniform flow of velocity \(V\) in a fluid of density \(\rho\) is given by

\[
L = \rho V \Gamma. \quad (1)
\]

The symbol \(\Gamma\) represents what is called the circulation about the object. It is found by erecting an imaginary circuit which completely surrounds the object, calculating at each point on that circuit the multiple of the flow velocity component in the circuit’s direction and the circuit’s length element over which it acts, and then summing all such multiples around the circuit. Both Bryant and Williams\(^{(2)}\) and Tanner\(^{(3)}\) carried out this laborious calculation using their experimentally obtained flow velocities. Using equation (1), the results they obtained for their aerofoils’ lifts were extremely close to those calculated from the measured pressure distributions; in the Bryant and Williams\(^{(2)}\) case the difference was 6%, for Tanner\(^{(3)}\) a mere 2%. The conclusion to be drawn from this is that an aerofoil experiencing a lifting force must also be experiencing circulation in its surrounding flow field.

One flow field which clearly possesses circulation is the vortex (see Appendix 1) shown in Figure 5. This also provides a gratifyingly simple illustration of the circulation calculation since on each circular streamline the flow velocity, \(v\), remains constant. It turns out that the velocity \(v\) is related to the streamline’s distance from the vortex centre, \(r\), by the relation

\[
v = k/r \quad (2)
\]

where \(k\) has a constant value throughout the vortex. To calculate the circulation, we choose any circular streamline as our circuit and multiply \(v\) (constant and in the direction of the streamline around this circuit) by the circuit length over which it acts, which here is the complete circumference \(2\pi r\). By equation (2)

\[
\Gamma = v \times 2 \pi r = 2 \pi k. \quad (3)
\]

Thus this vortex produces a constant circulation whatever the circuit selected about its centre.
It is therefore plausible to see the lifting aerofoil of Figure 3 as possessing a finite circulation around itself which is due to a clockwise rotating vortex superimposed on the whole flow field. Such a vortex would then produce the upflow / downflow described earlier as well as the higher velocities on the upper surface and reduced velocities on the lower. Indeed, this view of the lifting aerofoil now forms the basis of what is called the circulation or vortex theory of lift (see Appendix 3) and aerodynamicists talk of this vortex as the ‘bound’ vortex constantly surrounding the aerofoil. The question then becomes: how does a lifting aerofoil continuously manage to induce a vortex-like circulation around itself?

![Figure 5 Vortex flow.](image)

The first step in answering that question is to introduce another of those theorems which make aerodynamics such a thoroughly mathematical subject. This theorem is usually referred to as Stokes’ theorem (see Appendix 4) and it insists that a flow field possessing circulation must also possess, within the circulation circuit, fluid elements which are spinning. The formal terminology for this spinning effect is vorticity, which is the rate of spin. Figure 6 shows such a fluid element, taken to be circular in shape. The weight of this element acts through its centre as do the pressures acting around its periphery. Therefore neither of these types of forces can cause the element to spin. The only forces capable of causing spin or vorticity are the shearing forces shown, and these are created by viscous action.

![Figure 6 Forces on a cylindrical fluid element](image)

It is commonly accepted that oil and treacle, for example, exhibit viscous behaviour. It is perhaps far less widely known that all fluids – gases, liquids, even molten metals – possess some degree of viscosity. However, the coefficient of viscosity varies enormously from fluid to fluid. Air, for example, has a viscosity coefficient which is one hundred thousandth ($10^{-5}$) that of engine oil.

Viscous action is created by the random motions of the molecules in a flowing fluid: molecules from a higher velocity layer collide with those from layers of lower velocity and vice versa, thus creating momentum exchanges which appear as shearing forces between layers. In addition, such random molecular motions lead to a further effect when molecules impinge on solid surfaces. The fluid molecules become stuck there until kicked out by further molecular collisions, after which they depart in random directions. Thus at the macroscopic level at which we view flows, the fluid at the surface is seen to be at rest relative to that surface, an effect known as the no-slip condition.
The consequence of the above is that, when a fluid of very low viscosity such as air or water flows over a solid body, the no-slip condition combined with viscous action creates a layer of slower moving flow around the body in which fluid elements possess intense vorticity. As a rough analogy, you can think of this part of the flow as being composed of myriad tiny ball-bearings spinning away as they move more sluggishly around the body. But as seen in Figure 7, whereas there is clockwise vorticity on the aerofoil’s upper surface, on the lower surface the vorticity is in a counter-clockwise sense. However, the flow being faster on the upper surface, the total clockwise vorticity there is greater than that in the counter-clockwise sense on the lower surface. According to Stokes’ theorem, for which we take clockwise circulation and therefore clockwise vorticity as positive, the net positive vorticity within this heavily viscous layer is equal to the lifting circulation about the body.

This heavily viscous layer is known as the boundary layer and it is often exceedingly thin, usually a matter of a few millimetres in thickness. Through it, the flow velocity increases rapidly from the above-mentioned zero value relative to that surface to the velocity values experienced, for example, in the wind tunnel flow depicted in Figure 3. Within this thin layer the fluid not only possesses intense vorticity but is also subject to strong viscous shearing stresses. The shearing stress actually at the surface itself thus creates what is more commonly known as skin friction drag. In the flow field exterior to the boundary layer, fluid elements also experience shear stresses but here they are negligibly small. That part of the flow, the vast majority, can then be treated as an inviscid, non-spinning or irrotational flow field for which much simpler mathematical methods are available for its calculation (see Appendix 2).

The above argument depends heavily on the good behaviour of the boundary layer; it must remain very thin and attached to the body’s contours until the sharp trailing edge is reached. It is the aerofoil’s streamlined shape which enables the boundary layer to do this. At the trailing edge, however, the boundary layer, unable to flow around this sharp edge, finally separates to form a thin, slower moving wake. Figures 3 and 4 suggest that this is occurring in Tanner’s case (3). Thus it is that the streamlined aerofoil, through its enforcement of separation solely at the trailing edge, through viscous action creates a fixed and steady amount of fluid vorticity in its boundary layers, which in turn generates the overall circulation and hence lift. We fly, therefore, courtesy of the air’s very small viscosity, a fact which seems near-miraculous when we consider that even the smaller modern airliners possess weights equivalent to a dozen or more double-decker buses.

The announcement of the boundary-layer concept by Ludwig Prandtl (1875-1953) in 1904 is described in Appendix 5, together with further details of the layer’s behaviour. However,
two basic points concerning the latter should be made here. The first concerns the discrepancy between the theoretical and the observed pressures seen at the aerofoil’s trailing edge of Figure 4. The slower moving boundary-layer flow effectively shoulders aside slightly the inviscid flow outside the boundary layer. The consequence is that the pressure in the exterior flow does not quite return to the theoretical value at the trailing edge, as seen in that figure. This results in a slight pressure drag on the aerofoil in addition to that caused by skin friction.

The second point concerns the case in which an aerofoil’s incidence is set so high that upper surface suction becomes too deep. In that case the subsequent pressure rise over the aft part of the aerofoil becomes too rapid and the boundary layer separates at some point on that surface. The result is aerofoil stall, a dangerous condition entailing a loss of lift which can be large and quite sudden (see Appendices 5 & 7).

Thus far the flow fields described have been those about aerofoils, in the case of Figure 3 a length of wing which has constant shape and size set between the walls of a wind tunnel. This can be seen as a length taken from a wing of the same sectional shape but which has an infinite span. For the practical situation of the aeroplane in flight, in contrast, its wing has a finite span, its planform can vary in shape and it possesses tips.

For the wing of finite span the question arises: what happens to the bound vortex, and its circulation, at the wing’s tips? The answer depends here again on a further mathematical theorem due originally to Hermann von Helmholtz (see Appendix 4). This requires that a vortex, once created, cannot end and that its circulation must in some way be preserved. The smoke ring provides a good example; its vortex structure never ends since it exists as a closed toroidal loop having the same circulation all the way around.

In the case of the finite-span wing, the bound vortex becomes cast off progressively toward the tips, the released pair of vortices turning themselves roughly through right angles so as to spin away downstream whilst rotating in opposite senses (Figure 8). And this vortex system never ends. These so-called trailing vortices can be traced right back to the airfield at which the wing first generated lift. There the vortex loop is finally closed by the starting vortex which is left behind at the airfield. The circulation of this vortex system is preserved all the way around the loop so that the starting vortex has a spin sense opposite to that of the wing’s bound vortex. In fact, this starting vortex is generated by the wing’s sudden shedding of a part of its lower surface boundary layer’s vorticity at the instant at which the wing’s incidence is increased so as to generate its own lifting vortex. At that instant, the boundary layer adjusts so as to
continue to insist upon separating at the sharp trailing edge.

The trailing vortices behind high-flying jet aeroplanes are now a familiar sight. Their distinctive white vapour initially emanates from each engine exhaust, the result of the combustion of hydrocarbon fuel in the air’s oxygen. A combustion product is therefore hydrogen oxide, more commonly known as water and here in its vapour phase. The exhausts from, for example, a four-jet airliner become swept into the trailing vortices, the result being that each wing’s two exhausts coalesce to leave the familiar pair of white trails persisting for miles.

The manner in which a wing’s bound vortex is cast off depends on the wing’s planform shape. This process leads to a modification of the lift produced by the wing, the lift now being dependent on wing aspect ratio (the ratio of the wing’s span to its average chord). Moreover, as a wing flies forward it leaves behind two trailing vortex contributions which increase continuously with time. And these additional swirling motions possess kinetic energy. There is a basic axiom of dynamics which requires that, in creating additional kinetic energy, work must be performed – a force must move through some distance. In the case of the wing, a continuous propulsive thrust is required to push the wing forward so as to pump additional kinetic energy rearwards. But since the wing moves forward at constant speed, this propulsive force must be opposed by an equal but reactive drag force. This additional drag force is referred to as induced drag, the drag induced by the pair of trailing vortices which the wing cannot escape continuously creating.

An aeroplane’s total drag force, the sum of induced, skin friction and pressure drags, can be minimized by judicious choice of wing aspect ratio and the shapes of the aerofoil, wing planform and fuselage whilst ensuring that the lift is high enough for the aeroplane’s mission requirements.

3. **How the Correct Theory of Flight Came to Britain**

The paper by Bryant and Williams \(^{(2)}\) of 1924 appeared toward the close of a short period during which Britain had rapidly acquired the key ideas in the correct understanding of flight. Prior to around 1919, in marked contrast, little of this was known to the British scientists and engineers involved in aeronautics. However, there was one notable exception, the British automobile engineer Frederick William Lanchester (1868-1946) who, rather as a hobby, had taken up the study of flight in the early 1890s.

In 1907 Lanchester published his thinking on the subject in his book, *Aerodynamics* \(^{(5)}\). The book demonstrates that he had independently arrived at the basic concept of the boundary layer, which he called the ‘inert layer’, and with this he was able to provide realistic assessments for skin friction drag and its variation with flow velocity. He also advocated the use of streamlined shapes about which the ‘inert layer’ would be, as he put it, ‘conformable’, so that separation would be avoided up to the trailing edge. In addition, he had independently reached the conclusion that a lifting wing generates trailing vortices and suspected that it must therefore be experiencing circulation. From this he was able to calculate wing lift and induced drag; his results, although not accurate by modern standards, show the correct trends in regard to
the dependence on wing aspect ratio (see Reference 6). For most of his British contemporaries, however, his arguments on wing theory in particular were difficult to understand, his methods were judged to lack cohesion whilst his mathematical analyses lacked rigour; in short, his work was easy to criticise and, for the most part, ignore. In 1908 Lanchester visited Göttingen University, there to discuss a German translation of *Aerodynamics* with the English-speaking mathematician Carl David Tolmé Runge (1856-1927). And there he met Prandtl, who had already gained a better grasp of the boundary-layer concept and had begun to work on wing theory. In a lecture to this Society in 1927, Prandtl (7) described his exposure to Lanchester’s wing theory with carefully precise honesty: “In England you refer to it as “the Lanchester-Prandtl theory”, and quite rightly so, because Lanchester obtained independently an important part of the results. He commenced work on it before I did, and this no doubt led people to believe that Lanchester’s investigations, as set out in 1907 in his “Aerodynamics”, led me to the ideas upon which the aerofoil theory was based. But this was not the case. The necessary ideas upon which to build up that theory, so far as these ideas are comprised in Lanchester’s book, had already occurred to me before I saw the book. In support of this statement, I should like to point out that as a matter of fact we in Germany were better able to understand Lanchester’s book when it appeared than you in England. English scientific men, indeed, have been reproached for the fact that they paid no attention to the theories expounded by their own countryman, whereas the Germans studied them closely and derived considerable benefit therefrom. The truth of the matter, however, is that Lanchester’s treatment is difficult to follow, since it makes a very great demand on the reader’s intuitive perceptions, and only because we had been working on similar lines were we able to grasp Lanchester’s meaning at once. At the same time, however, I wish it to be distinctly understood that in many particular respects Lanchester worked on different lines than we did, lines which were new to us, and that we were therefore able to draw many useful ideas from his book.”

Yet Lanchester was not totally ignored by his aeronautical compatriots. He was appointed to the Government’s Advisory Committee for Aeronautics (ACA) at its formation in 1909. His work there covered a number of important issues: examples include skin friction calculations, engine design and performance (particularly at high altitude) and the rectification of the earliest British case of aerelastic instability (the ‘terrifying’ oscillations of the tail unit of the Handley Page O/100 bomber) described in Reference 8. However, as Jarrett (9) points out, Lanchester’s involvement in the design and construction of full-scale aeroplanes ended in failure. In marked contrast to his extraordinarily light-weight flying models, these aeroplanes were over-engineered, introduced too many untried innovations and had excessively substantial structures in comparison with the light-weight simplicity adopted by pioneers such as the Wright brothers.

Between 1909 and the onset of the First World War the ACA included with its reports authoritative reviews of papers on aeronautics which had been published abroad. Yet in these there is scarcely a mention of the seminal ideas emerging on the boundary-layer and lift concepts of Prandtl, Kutta and Zhukovskii. After the onset of the War, such reviews ceased. Perhaps due to wartime exigencies, workers at the RAF, Farnborough, and the Aerodynamics Department of the National Physical Laboratory (NPL), lacking any guidance from those concepts, appeared to take the view that a wing’s lift was a phenomenon best investigated
solely by experiment. Consequently, various aerofoil shapes and wing planform arrangements were examined in numerous wind tunnel investigations. From these various rules-of-thumb for shape characteristics were proposed, commensurate with the need to accommodate the conventional twin-spar biplane wing structures. But lacking the basic concepts described in Section 2, it was difficult to achieve a deeper understanding of the results obtained. On the influence of wing aspect ratio, for example, Ernest Frederick Relf (1888-1970) at the NPL noted \(^{(10)}\) in 1918 that, although most of the wing tests had been conducted with an aspect ratio value of 6 as standard, from the few other results available he could go no further than to suggest that “there is a general tendency for the lift coefficient to rise and the drag coefficient to fall with increasing aspect ratio,” (see Appendix 6 for definitions of such coefficients). Both of these tendencies had been predicted in Lanchester’s calculations \(^{(5)}\) which, by 1918, had been given greater substance by Prandtl.

Britain’s continuing problem caused by a lack of basic concepts was at the root of an argument which developed between Farnborough and the NPL concerning the accuracy of wind tunnel data. Farnborough doubted that the NPL’s data obtained from tests on small-scale models could realistically be applied to full-scale work at the RAF. This argument on what became known as ‘scale effect’ resulted in the formation of a committee in 1917 to investigate the matter. In this one senses the dawning realisation that here some crucial concept was being missed. The dilemma faced by the committee is highlighted by the case of the Fokker wing tests reported in 1919 (see Appendix 7).

Occasionally, however, there was evidence of deeper insights in Britain. One example is provided by the wind tunnel investigation \(^{(11)}\) of the downflow behind a lifting wing conducted by Norman Augustus Victor Piercy (formerly Tonnstein) (1891-1953) at the East London College (later, Queen Mary College, University of London). He remarked that this downflow was similar to that calculated by Lord Rayleigh from his analysis of a rotating cylinder experiencing circulation (see Appendix 3), and “which has been applied to the case of aerofoils by Lanchester”. A more telling example is the wind tunnel investigation \(^{(12)}\) by Geoffrey Ingram Taylor (1886-1975) at Farnborough to determine the skin friction drag on a flat plate at zero incidence. Rather than measure this force directly, he measured the flow velocity variations close to the plate’s surface using a fine Pitot tube (see Appendix 2). He found that the velocity increased rapidly from zero at the plate’s surface to the tunnel’s free flow value. To calculate the drag, he used his measurements in conjunction with his analysis of the momentum degradation in this slower moving flow. That analysis produced an equation similar to a more general equation, the boundary-layer integral momentum equation, published by Prandtl’s former doctoral student, Theodore von Kármán (1881-1963), in 1921 \(^{(13)}\).

However, in his report \(^{(12)}\) Taylor includes a plea that the NPL “may see its way to repeat the experiments with the more adequate resources which they have at their disposal.” He adds, “The importance of the results which might be obtained is connected rather with the theory of fluid friction than with any direct practical application, but on the other hand our lack of knowledge of the mechanism by which surface friction acts is the chief difficulty in any attempt at a theory of the flow of a fluid past a solid body.” At the time, this recommendation to investigate what Taylor \(^{(12)}\) called the “surface layer” and thereby acquire the boundary-layer concept seems to have been disregarded. Had that not been so then, the crucial element
missing from the debate concerning ‘scale effect’ mentioned above would have clarified the matter and explained the strange results in the Fokker aerofoil tests described in Appendix 7.

In Germany during the wartime period, in contrast, Göttingen was forging ahead in the understanding of boundary-layer behaviour and in the development of finite wing theory based on the circulation concept introduced by Kutta and Zhukovskii. As to the latter’s work, in 1915 Albert Betz (1885-1968) published his experimental results \(^{14}\) for the pressure distribution around a Zhukovskii aerofoil, results which showed good agreement with Zhukovskii’s theory. As to finite wing theory, an early application \(^{15}\) came from Carl Wieselsberger (1887-1941) in 1914, in which he showed why it is that Zugvögel (migrating birds) tend to fly in V-formations (Figure 9). The trailing vortices of the foremost bird assist its two immediate followers and so on down the formation. This paper, now one hundred years old, has recently excited interest by raising the possibility that long-distance airliners might save fuel by flying in formation. However, the culmination of Göttingen’s wartime research on finite wings came in two reports \(^{16, 17}\) by Prandtl in 1918 and 1919.

\[\text{Figure 9} \quad \text{The V formation of migrating birds} \quad (\text{Wieselsberger}^{15}, \ 1914).\]

by following a curve of elliptic shape. The easiest means by which this can be achieved is to select an elliptic wing planform, a feature suspected by Lanchester \(^{5}\) in 1907 as producing minimum induced drag and used by him in a number of his flying models. However, in his report \(^{16}\) Prandtl chose to illustrate this idea not with a simple ellipse but with a combination of two semi-ellipses (Figure 10). This planform is geometrically identical to that adopted in the design of the Supermarine Spitfire of 1936; the possible connection here is traced in References 18 and 19.

The situation in Britain changed dramatically shortly after the War as the result of a visit to Göttingen by Ronald McKinnon Wood (1892-1956), then the head of Farnborough’s Aerodynamics Department. He took with

\[\text{Figure 10} \quad \text{The elliptic wing} \quad (\text{Prandtl}^{16}, \ 1918).\]
him his colleague and accomplished mathematician, Hermann Glauert (1892-1934); as Brinkworth’s extensive review \((20)\) of research on an aeroplane’s spinning motion emphasises, Glauert was already noted for his analysis of this then highly dangerous phenomenon. The purpose of their visit was to view Prandtl’s new wind tunnel, operating since 1917 and known to produce superior flow quality. Glauert came from a family of German immigrants established in the cutlery business in Sheffield and was fluent in German. He returned to Britain not merely with the details of Prandtl’s wind tunnel but, more importantly, with Göttingen’s work on boundary layers and the theory of the finite wing. His review \((21)\) of this, whilst including a brief description of the boundary layer, concentrated mainly on wing theory. His subsequent career in which he introduced improved mathematical methods to wing theory, extending this to a wide variety of aerodynamic problems, is described in Reference 22.

One of Glauert’s great contributions was in the field of aeroplane stability theory. This had been put on a firm footing by George Hartley Bryan (1864-1928) in his book, *Stability in Aviation* \((23)\), published in 1911. A description of Bryan’s less-than-happy career can be found in Reference 24. However, Bryan’s stability theory arrives at six equations - one for each axis about which an aeroplane can rotate and one for each motion in those axis directions - which describe motions slightly disturbed from equilibrium. Each equation has four terms, the coefficients of which have, by some means, to be known in order to determine whether the disturbance decays, so that the aeroplane is stable, or grows to instability. The coefficients represent specific out-of-balance forces and moments afflicting the disturbed aeroplane: examples are the rolling moment due to rate of roll, the yawing moment due to rate of yaw, and so on. There are then twenty-four such coefficients and the problem by the close of the War was determining them. Some could be calculated using concepts by then understood. Yet others, for want of theoretical guidance, could not and there seemed no option but to measure them in wind tunnel experiments. This would have become a substantial task considering that such tests would have to be performed for each and every aeroplane type. Luckily, Glauert was able to show that many of these coefficients could be calculated using the newly acquired wing theory.

Thus in the early 1920s Britain rapidly began to catch up. In 1921 Muriel Barker (1892-1949) (later to become Mrs Glauert) produced a report \((25)\) on Zhukovskii aerofoil sections, following this in 1923 with a lecture \((26)\) to this Society on more recent developments in aerofoil theory. In the same year Piercy \((27,28)\) published his results for two experimental investigations, the first \((27)\) involving measurements of the trailing vortex structure behind a lifting wing in a wind tunnel. He confirmed that the vortex’s outer structure agreed with that shown in Figure 5. However, viscous action dominated in the vortex’s central core so that the air was there effectively in solid body rotation and contained intense vorticity; this, by Stokes’ theorem, was at the root of the trailing vortex’s circulation. His second investigation \((28)\) involved a detailed study of the boundary-layer development over an aerofoil’s surface in which he measured the velocity variation through the layer. He was also able to confirm a basic result obtained by Prandtl that the local pressure does not change through the layer.

As to boundary-layer theory itself, this received attention in a lecture \((29)\) to this Society in 1924 by Leonard Bairstow (1880-1963), formerly head of the NPL’s Aerodynamics
Department and by then Zaharoff Professor of Aviation at Imperial College, London. Bairstow presented his approximate theory for the boundary-layer development on a flat plate, showing that his results compared favourably with those of higher accuracy obtained by Prandtl’s doctoral student, Heinrich Blasius (see Appendix 5), which had been published in 1908. During the discussion after the lecture’s close, Bairstow received sharp criticism from Archibald Reith Low (1878-1969), often a thorn in the side of British officialdom, the criticism here being directed at Britain’s neglect of both Lanchester’s and Prandtl’s ideas. This prompted Bairstow to admit that he “had heard of the boundary layer theory of Prandtl before 1914, but it was not until Easter of this year that I became aware of the solitary solution of Blasius.” Another contributor to the discussion went further. Richard Vynne Southwell (1888-1970) expressed “a personal sense of regret that he had been unaware of the work which Professor Bairstow had described for so long a time after its publication. It was necessary to face the fact that Prandtl’s boundary layer theory had been published in 1904, and Blasius’ calculations for the thin plate in 1907 [the year of his Göttingen thesis], so that both had been available for two years when the Advisory Committee for Aeronautics began its work in 1909! For his own ignorance, of course, no one was to blame but himself; but he would like to ask whether we were satisfied with the share which this country had taken in recent developments of fundamental hydrodynamic theory.”

At that time Southwell was the Superintendent of the NPL’s Aerodynamics Department. Despite his mea culpa above, he must have drawn some satisfaction from watching the experiments there by Bryant and Williams which provided verification of the fundamental theory of Kutta and Zhukovskii, specifically equation (1), and which was published \(^{(2)}\) in that same year of 1924. That the paper was then re-published in the Royal Society’s \textit{Philosophical Transactions} of January 1926 is an indication of the high importance attached to its findings. Publication by the Royal Society required that, even before peer review, a paper must be communicated, and therefore vouched for, by a Fellow. In this case the communicator was Bairstow, a Fellow since 1917.

As Britain at last took on board these new ideas in aerodynamics, one rightful consequence was that Lanchester finally received due recognition. He was elected Fellow of the Royal Society in 1922 (Prandtl was elected Foreign Member in 1928), this Society presented him with its Gold Medal in 1926 (Prandtl received this and Honorary Fellowship in 1927) and he was awarded the Daniel Guggenheim Medal in 1931 for his contributions to the fundamental theory of aerodynamics (Prandtl received this award in 1930 for his pioneer and creative work in the theory of dynamics).

Both Bairstow’s and Southwell’s comments suggest that British ignorance of crucial concepts devised abroad pre-dated the First World War’s onset and its exigencies. Having learnt its lesson, Britain subsequently kept a close eye on aerodynamic developments in Germany, particularly those at Göttingen. Glauert was assiduous in this respect until his untimely death in 1934 \(^{(22)}\). A small number of British aerodynamicists worked at Göttingen for short periods, gaining first-hand knowledge of its activities. Examples are:

- Sydney Goldstein (1903-1989) during 1928-1929, later to become the distinguished editor of Reference 4. In his subsequent career he produced important publications on aerofoil and boundary-layer theories, and on the nature of turbulence \(^{(30)}\).
• Herbert Brian Squire (1909-1961) during 1932, before beginning his notable career at Farnborough and Imperial College, London. His work covered a wide range of aerodynamic problems, most notably jets and helicopter rotor flows.\(^{(31)}\)

• In 1930 John Watson MacColl (1903-1977) provided this Society with an extensive review\(^{(32)}\) of current work at Göttingen and in von Kármán’s group at Aachen as a result of his visits there. His most notable publication\(^{(33)}\) was with G. I. Taylor on supersonic cone flow. His subsequent career was in armament research, much of it secret, at Fort Halstead, Kent, where he became Superintendent of Basic Research.

4. Concluding Remarks

It is clear that Britain came rather late to the correct understanding of flight. As to that widely popular but flawed explanation with which this article opened, it may be that it sprang up during the era prior to the end of the First World War in which it had no competition, no other explanation having gained acceptance in Britain.

This article includes condensed versions of my material contributed to Reference 34, the latter providing English translations of some of the papers by Prandtl, Kutta and Zhukovskii mentioned here. Aerodynamicists may well take the view that the result is an article which is too superficial: corners have been cut, restrictions have not been stated, and so on. For all of that I apologise but my aim is that readers new to the subject might get the gist of the matter. For further reading, there are nowadays a number of fine texts on the market, yet Glauert’s book\(^{(35)}\), first published in 1926, still provides an excellent basic-level introduction.

Acknowledgements

The author gratefully acknowledges the generous assistance of Brian Riddle, Librarian of the Royal Aeronautical Society, and Anthony Pilmer of the National Aerospace Library, Farnborough. Gratitude must also be expressed to Frank Armstrong FREng, FRAeS, Professor Peter Bearman FREng, FRAeS, Peter Laws MRAeS, Roger Nolte, Helen Outram, Professor Norman Riley and to Evolve Design Consultants for assistance with some of the graphics.

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Appendix 1

Continuity and Vortices: The Statements of Leonardo da Vinci

Truesdell (36) points out that Leonardo da Vinci (1452-1519) appears to be the first on record to state two principles which are fundamental to fluid flows. Presumably these were based on observation. The first concerns the continuity principle as applied to river flows (Truesdell’s suggested correction in brackets):

“If the water is not added to or taken away from the river, it will pass with equal quantities in every degree of its breadth [length?], with diverse speeds and slownesses, through the various straightnesses and breadths of its length.”

This fundamental principle, that the quantity of flow – basically the mass flow rate – must remain unchanged applies to all flows. Thus far, however, Leonardo’s statement had been implied in those by such Greek scientists as Hero of Alexandria. But then Leonardo adds:

“By so much as you will increase the river in breadth, by so much you will diminish the speed of its course.”

It is this statement of a direct, and correct, proportionality which is the crucial step, and it translates immediately to Section 2’s discussion concerning flow speed and the distance between the streamlines of Figure 3.

More fundamentally, imagine a group of streamlines bundled together to form a tube. By the very nature of streamlines, fluid can neither leak in nor leak out along this tube. If the cross-sectional area at any point in this tube is A and the flow velocity there is v, then the volume of fluid passing through A in unit time is vA. If the fluid density there is ρ then the mass passing through A in unit time, the mass flow rate, is ρvA and this must remain constant along the tube. For a fluid of constant density, it follows that v must vary inversely as A along the tube, as above.

The continuity principle was widely accepted once flow problems began to be investigated. For example, Isaac Newton (1642-1727) used it in his study of the efflux from the hole at the base of a water vessel which he describes in Principia (37). In the 18th century’s development of the field theory of fluid flow outlined in Appendix 2, the continuity principle was incorporated as a relationship which must always be obeyed in all flow calculations.

Leonardo’s second statement concerns vortex structure (Truesdell (36));

“The helical or rather rotary motion of every liquid is so much the swifter as it is nearer to the centre of its revolution...we have the same motion, through speed and length, in each whole revolution of the water, just the same in the circumference of the greatest circle as in the least...”

This is in agreement with the vortex equations (2) and (3) of Section 2, particularly with regard to the result that the circulation has the same value on each circular streamline.
Appendix 2

The Bernoulli Principle and the Field Theory of Fluid Flow

The Bernoulli principle has its origin in the investigations of the Swiss-born physiologist and mathematician Daniel Bernoulli (1700-1782) whilst working in St Petersburg. Using the continuity principle in conjunction with the ‘vis viva’ (kinetic energy) concept of Gottfried Wilhelm von Leibniz (1646-1716), Daniel produced analyses for flow situations such as that shown in Figure 11. In this he treated the water flow en masse, so to speak, and conceived of pressure only in terms of the height of the water column (manometer) shown in Figure 11. This height he was able to calculate and found experimental confirmation for his results. He published his findings in 1738 in his book, *Hydrodynamica* (38). In Basle, Daniel’s father Johann Bernoulli (1667-1748), Europe’s leading mathematician following Newton’s death, published his treatise on fluid flow, *Hydraulica* (39), in 1743, yet dated it 1732 in an evident attempt to steal his son’s thunder.

![Figure 11 Water efflux from a vessel (Bernoulli (38), 1738)](image)

For many years *Hydraulica* was dismissed as disgraceful plagiarism until the correspondence on the affair was examined by Truesdell (40) in 1954. It emerged from this that Daniel’s friend and Johann’s former pupil, Leonhard Euler (1707-1783), whilst commiserating with Daniel for his father’s treatment of him, had also written to Johann to congratulate him on the new ideas contained in *Hydraulica* – rightly so, since Johann’s analysis contained at least two new ideas which were significant advances. The first was that Johann had replaced Daniel’s
water-column measures of pressure by forces due to pressure within the flow itself. Secondly, having recognised that flow conditions must vary continuously along pipes of varying area according to the continuity principle, Johann had then taken the vital step of analysing conditions at an elementary slice of fluid across the pipe. Moreover, the physical principle which Johann had then applied to his moving fluid slice had no longer been his previously preferred ‘vis viva’ concept but Newton’s version of the Second Law of Motion stated in *Principia* (37). After further analysis using the powerful tool of integral calculus, Johann’s result was recognisably close to the modern statement of the Bernoulli equation, which we will come to presently. It was this combination of a focus on an infinitesimally small fluid element coupled with the application to it of the Second Law which Euler promptly seized upon, generalized, and exploited with dramatic success.

Added to the above was Euler’s realisation that Newton’s statement of his Second Law in *Principia* (37) was not quite how he had applied it in his own analyses contained in that work. This ‘detective work’ on Euler’s part resulted in his paper (41) of 1750 in which he asserted as “the unique foundation of all mechanics” the principle that force is proportional to the product of body mass and acceleration, thus giving us the modern version of the Second Law. Having also gained a correct conception of fluid pressure, Euler brought all these ideas together to construct the equations of continuity and motion for a field of flow containing continuous variations of pressure and flow velocity. Detailed commentaries on Euler’s papers can be found in Truesdell (40), showing that from these the modern version of the Bernoulli equation finally emerged. This states that along any streamline within an inviscid, irrotational flow which does not vary with time (steady flow), the fluid having a constant density $\rho$, the pressure $p$ and flow velocity $v$, are related by the equation

$$p + \frac{1}{2}\rho v^2 = \text{constant} \text{ along the streamline.} \quad (4)$$

Thus an increase in flow velocity along a streamline necessarily results in a decrease in pressure, and vice versa, as argued in Section 2 with regard to Figure 3. In that case, if we trace a streamline back to the uniform wind tunnel stream approaching the aerofoil where the pressure is $p_0$ (called the free stream static pressure) and the tunnel velocity is $U_0$, then

$$p + \frac{1}{2}\rho v^2 = \text{constant} = p_0 + \frac{1}{2}\rho U_0^2. \quad (5)$$

And if we apply this equation to the streamline which attaches to the aerofoil just beneath its nose, the flow coming to rest there at the stagnation pressure $p_{St}$, then

$$p_{St} = p_0 + \frac{1}{2}\rho U_0^2. \quad (6)$$

To return to Euler, his papers published between 1752 and 1766 showed that for inviscid, irrotational flows the governing equations reduce to a form usually associated with the later work of Pierre-Simon, Marquis de Laplace (1749-1827). Euler pointed out that this equation has simple solutions which represent the flow fields shown in Figure 12: the uniform stream, the source, the sink, and the vortex, the latter being that shown in Figure 5 and described by equation (2). Euler also showed that, due to the mathematical properties of this ‘Laplace Equation’, all such simple solutions are additive in the sense that their flow velocities can be
added vectorially so as to obtain more complicated flows. Later mathematicians realised, for example, that a source combined with a sink of equal but opposite strength produces the flow field shown in Figure 13a. Add to that a uniform stream and we obtain Figure 13b. Here you see that all of the source/sink flow is contained within an oval streamline which can be taken to represent the surface of a solid body. If, under certain mathematical conditions, the source and sink of Figure 13b are brought together to form what is called a dipole or doublet, then we obtain the flow about the circular cylinder shown in Figure 14a. Add to that a vortex centred at the cylinder’s centre and we obtain Figure 14b representing a circular cylinder experiencing circulation.

![Flow Fields](image)

**Figure 12** Irrotational flow fields: (a) uniform stream, (b) source, (c) sink, (d) vortex.

Euler’s contemporary, the French mathematician Jean Le Ronde d’Alembert (1717-1783), also attempted a field theory of inviscid flow but with less success. Nonetheless, in 1752 he obtained the result\(^{(42)}\) that, for steady flows, the drag force on a body is precisely zero, a result which Euler had already obtained in 1745. Revisiting this result in 1761, d’Alembert\(^{(43)}\) could still go no further than to leave it as a “paradox proposed to the geometers on the resistance of fluids.” That the result is nonetheless correct can be seen from the streamline patterns shown in Figures 13b and 14 for bodies which are symmetric fore and aft. In all of these cases the patterns are also symmetric fore and aft and therefore so must be the pressure distributions. Indeed, this result holds for all finite body shapes, symmetric or not.

Subsequently, d’Alembert’s Drag Paradox, as it came to be known, bedevilled the application of field theory to flows until the role of viscosity came to be understood. Until that time, practical men looked askance at this new field theory, seeing it as a mathematical abstraction of little value. Nonetheless, it was the streamline pattern of Figure 14b, with its top-to-bottom asymmetry, which provided the key to the lift theory outlined in the next appendix. And once the viscous boundary layer’s behaviour on well-streamlined bodies had been
understood, it was seen that the field theory was very effective in describing the rest of the flow.

Figure 13  (a) Flow field of source/sink pair  
(b) source/sink pair combined with uniform stream.

Figure 14  (a) Flow about circular cylinder  
(b) flow about circular cylinder with vortex centred at cylinder’s centre.

It is possible to generate any shape of body by distributing suitable combinations of sources, sinks and doublets within a uniform stream, the only restriction being that the total source and sink strengths must be equal in order to obtain a body having a closed contour. With that approach, streamline patterns and pressure distributions can be calculated. However, an alternative approach is suggested by the similarity of the flow patterns shown in Figures 12 and 13a to the flux lines in electric and magnetic fields; the source and sink of Figure 12 resemble North and South poles, and so on. In fact this resemblance is no mere chance since all such field cases are governed by Laplace’s equation and therefore possess the same mathematical solutions. This analogy was exploited by Relf \(^{(44)}\) in 1924 so as to devise an apparatus which generated an electric field about the body shape of interest. This enabled inviscid flow streamline patterns to be plotted, for example, by Bryant and Williams \(^{(2)}\) for the
aerofoil shape used in their investigation. This method might also have been used to obtain the theoretical streamlines shown in Figure 3, although this is not stated in Reference 4 from which that figure is taken.

Daniel Bernoulli is often credited as being the first to use manometers for the measurement of flow pressure. However, six years before the publication of Hydrodynamica, Henri Pitot (1695-1771) had given details of his experiment to measure the speed of the River Seine in Paris. For this he had used two open-ended manometer tubes inserted vertically into the flow. One tube was straight and, as we now understand it, the water’s height in it provided a measure of the static pressure at the depth at which the lower end was placed. The other tube was lowered to the same depth but this tube had its lower end bent through a right angle with its opening set pointing into the river stream; the height of its water column therefore registered the stagnation pressure. Pitot found that the difference in heights registered by his manometers was proportional to the square of the river’s speed. Equation (6) derived from the Bernoulli equation gives precisely this result.

By 1912 in Britain the combined Pitot-static tube shown in Figure 15 was being used in aeronautical experiments. It consisted of two concentric tubes as show, the open-ended inner one registering stagnation pressure whereas the outer tube, with its closed nose, registered static pressure through the fine holes drilled around its surface. The difference in the two readings, by equation (6), provided a measure of \( \frac{1}{2} \rho U_0^2 \) in wind tunnel experiments or, for aeroplanes in flight, flight speed with the instrument connected to an airspeed indicator.

![Figure 15](image-url)

Figure 15 Pitot-static tube (Bramwell, Relf & Fage, 1912).
Appendix 3
The Circulation Theory of Lift

From antiquity, man-carrying flight had been a frequent ambition, yet little progress could be expected until Newton\(^{(37)}\) had set down the first steps toward our modern grasp of dynamics. And as those steps emerged, the Royal Society in London was providing a forum in which many physical phenomena could be discussed in a rather more rational manner. Indeed, as Lorenz relates\(^{(47)}\), in a little-known document of 1691 Newton’s contemporary Edmond Halley (1656-1742) provided that Society with an interesting and reasoned assessment of the wing area needed to support a man in flight, an assessment which arrived at a realistic figure for this.

Even so, little progress was achieved until Sir George Cayley (1773-1857) announced his aeroplane concept in the first decade of the 19th century\(^{(48)}\). In this he took the crucial step of separating the lifting and propulsion systems, thus doing away with the previous, invariably unsuccessful, flapping wings used in attempts to emulate bird flight. In his description of the airflow about an arched or cambered wing, Cayley\(^{(49)}\) suggests that the air “current is constantly received under the anterior edge of the surface, and directed upward into the cavity… The fluid accumulated thus within the cavity has to make its escape at the posterior edge of the surface, where it is directed considerably downward…” Cayley\(^{(49)}\) also mentions the possibility of “a slight vacuity” on the forward upper surface. These statements are some of the earliest to demonstrate an awareness of the upflow/downflow and upper surface suction phenomena mentioned in Section 2. Moreover, Cayley\(^{(50)}\) stressed the need for streamlined shapes in the pursuit of low drag, contrary to the then widely-held belief that the forward surfaces of a body are alone responsible for drag creation.

Despite the efforts of Cayley and others, however, many scientists continued to have little faith in the possibility of man-carrying flight. As late as 1896 Lord Kelvin declined an invitation to join the Aeronautical Society in London, stating\(^{(51)}\) that “I have not the smallest molecule of faith in aerial navigation other than ballooning or of expectation of good results from any of the trials we hear of.” Evidently he remained unimpressed by the widely-publicised achievements of the hang-gliding pioneer, Otto Lilienthal (1848-1896), in Germany. Lilienthal’s fatal gliding accident in that same year might have affected Kelvin’s opinion but, luckily, other scientists took a rather more sanguine view, as we shall see.

It can be argued that our understanding of aerodynamic lift began, not from grand ambitions to achieve flight, but from the rather more prosaic desire to explain the sideways curving trajectory of the sliced tennis ball. In a letter to the Royal Society in 1671, Newton\(^{(52)}\) drew attention to this phenomenon, using it as an analogy in his argument on the refraction of light: “…I remembred that I had often seen a Tennis ball, struck with an oblique Racket, describe such a curve line. For, a circular as well as a progressive motion being communicated to it by that stroak, its parts on that side, where the motions conspire, must press and beat the contiguous Air more violently than on the other, and there excite a reluctancy and reaction of the Air proportionally greater.”
Benjamin Robins (1707-1751) attributed the wayward motions of cannon and musket shot to this same spinning effect caused, he claimed in this case, by irregularities in gun barrels. In 1746 he demonstrated (53) the effect before the Royal Society using a wooden pendulum bob suspended by a double string “about eight or nine feet long”. By twisting the strings together, so that the bob spun about its stringwise axis, the pendulum motion was seen to include a sideways motion. Its direction he predicted correctly as being towards the bob’s side “on which the whirl would combine with the progressive motion; for on that side always the deflecting power acted; as the resistance was greater here, than on the side, where the whirl and progressive motion were opposed to each other.”

It was Heinrich Gustav Magnus (1802-1870) in Berlin who demonstrated that this “deflecting power” was due, not to Robins’ vague notion of “resistance” differences, but to pressure differences caused by the spinning motion. In 1852 he published a paper (54) describing his experiment in which an air jet had been directed at a spinning cylinder, the pressure variations being detected by movable vanes. When the cylinder rotates, as he put it, “the velocity, and consequently the decrease of pressure is greater on the side which moves in the same direction as the current than on the other side, where these motions have opposite directions.” Subsequently the phenomenon was named the Magnus Effect.

In 1877 Lord Rayleigh (1842-1919) referred to the Magnus experiment in his analysis (55) of the phenomenon noted by Newton, the curving motion of the sliced tennis ball. Rather than analyse the more complicated case of a sphere, however, he chose the circular cylinder flow of Figure 14b with its vortex centred at the cylinder’s centre. He referred to the fluid as “perfect…without molecular rotation”; in other words, he took the fluid to be inviscid, the flow irrotational and could therefore apply field theory to the problem. As to the generation of this vortex in practice, he suggested that if the cylinder rotates “the friction between the solid surface and the adjacent air will generate a sort of whirlpool of rotating air, whose effect may be to modify the force due to the stream.” Using the field theory outlined in Appendix 2, he calculated the flow velocities and cylinder pressure distribution to obtain a result for the cylinder’s lateral force which is, in effect, a statement of the later Kutta-Zhukovskii theorem (equation (1)) for this particular case.

It was the successful hang-gliding exploits of Lilienthal which prompted two mathematicians to attempt theoretical analyses of the flow about his fabric-covered wings formed into arched or cambered sectional shapes. Both analyses were based on the flow field of Figure 14b used by Lord Rayleigh. The first (56) was published in 1902 by Martin Wilhelm Kutta (1867-1944) who had already gained his doctorate in 1900 at the Technische Hochschule, Munich, for his subsequently widely-used work on the numerical solution of differential equations. At the instigation of his Munich colleague Sebastian Finsterwalder (1862-1951), who was interested in the scientific problem posed by Lilienthal’s flights, Kutta decided to base his Munich habilitation thesis on the Lilienthal wing problem.

In his analysis, Kutta (56) used a mathematical technique called conformal mapping to transform the cylinder flow field of Figure 14b into the flow about the aerofoil shaped as a circular arc shown in Figure 16. This is set at zero incidence, the only case published by Kutta (56) at this stage. The flow field shown is one of an infinite number of possibilities.
since any value can be selected for the vortex strength (essentially the value of k chosen in equations (2) & (3)). However, all but one value of the vortex strength result in a streamline departing from the aerofoil at some point on its surface other than its trailing edge, the flow whipping about that edge with infinite velocity and thus creating an infinitely negative pressure. Kutta\(^{(56)}\) therefore chose the one realistic case, Figure 16, in which the departing streamline leaves smoothly at the trailing edge and with a finite velocity. This selection therefore fixes the vortex strength, the circulation and hence the lift value. Put in terms of \(C_L\), the non-dimensional lift coefficient (see Appendix 6), his result for the lift per unit span, \(L\), is

\[
C_L = 4\pi \frac{h}{c}, \quad \text{where} \quad C_L = \frac{L}{\frac{1}{2}\rho V^2 c}.
\]  

(7)

Here \(\rho\) is the fluid density, \(V\) the free stream velocity, \(h\) and \(c\) the maximum height of the arched section and its fore-and-aft chord length respectively. On comparing his lift result with the experimental values obtained by Lilienthal\(^{(57)}\), Kutta\(^{(56)}\) found sufficiently good agreement to believe that he was on the right track.

To divert for a moment, Figure 16 provides a clear illustration of the fact that the popular explanation for flight described in Section 1 is flawed. Kutta’s aerofoil is experiencing lift and therefore a flow velocity on the upper surface higher than that on the lower. Using the element A/element B argument of Section 1, element A must reach the trailing edge before element B because, in this case, the path lengths to be traversed are identical.

To return to Kutta, in 1902 he had released his analysis\(^{(56)}\) for the single case of his aerofoil set at zero incidence. He delayed publishing his analysis for the finite incidence cases\(^{(58)}\) until 1910, probably because, in these cases, he had encountered a mathematical difficulty at
the sharp leading edge of his aerofoil. In the zero incidence case of Figure 16 the streamline approaching the leading edge attaches itself there. But in all of the finite incidence cases, whilst the departing streamline at the trailing edge is yet again arranged to leave smoothly there, thus fixing the circulation, the streamline which attaches itself to the forward part of the aerofoil does so at a point on the lower surface aft of the leading edge. The consequence is then that the flow whips around the sharp leading edge with infinite velocity. The panacea for this behaviour, Kutta \(^{(58)}\) suggested, was to blunt the leading edge slightly.

This difficulty was avoided altogether in the analysis of Zhukovskii \(^{(59)}\), also published in 1910, by adopting precisely that measure. But not only did he devise an aerofoil shape possessing a blunt leading edge but also one which had a streamlined contour. And the thickness distribution of this streamlined shape was added to the circular arc, the camber line of Kutta’s thin aerofoils of References 56 and 58. As in those cases, the ratio h/c, where h is the maximum height of the camber line’s arc and c is the aerofoil’s chord, denotes what is called the camber value. Figure 17 shows one of Zhukovskii’s aerofoils, this one possessing zero camber and which is therefore symmetric top-to-bottom.

Nikolai Egorovich Zhukovskii (1847-1921), Professor of Mechanics at Moscow Technical School, had visited Lilienthal in Berlin, had watched him glide and had purchased one of his gliders. Intrigued by these demonstrations of lifting performance, he too decided to provide a theoretical explanation for it. Like Kutta, Zhukovskii \(^{(59)}\) began his analysis with the cylinder-with-circulation flow field of Figure 14b, transforming it in his case into those about blunted-nosed streamlined shapes like that of Figure 17. At this stage, this transformation was accomplished by geometrical construction although later conformal mapping was used. In the notation used in equation (7), for aerofoils with small camber values, h/c, set at small incidences, \(\alpha\), a close approximation to Zhukovskii’s result \(^{(59)}\) is
This approximation also covers Kutta’s result\(^{(58)}\) of 1910 and it reduces to his earlier zero incidence result\(^{(56)}\), equation (7), when \(\alpha\) is zero. Given that this result holds only up to aerofoil stall (see Appendices 5 & 7), which usually occurs below an incidence of 15\(^o\) or so, for such small incidences it is legitimate to make the further approximation \(\sin \alpha \approx \alpha\) (in radians).

In achieving their results, both Kutta\(^{(56,58)}\) and Zhukovskii\(^{(59)}\) had derived the theorem, equation (1) of Section 2, which is now named after them. To fix their choice of vortex circulation for all incidences, they had both applied the condition of smooth flow at the trailing edge. This is now referred to as the Kutta-Zhukovskii condition. A physical explanation for why this is the correct choice was not immediately evident at the time, but the emerging boundary-layer concept was soon to provide this. Thus it was that the jigsaw pieces for the scientific explanation of flight finally began to fit together – and this at a time when the Wright brothers were demonstrating that powered, controllable flight was a reality. Sheer serendipity!

In 1910 both Kutta\(^{(58)}\) and Zhukovskii\(^{(59)}\) mentioned with approval Lanchester’s circulation theory of 1907 contained in his Aerodynamics\(^{(5)}\). Prandtl was therefore not alone in granting foreign approval to the book.

Equation (8) was later found to agree remarkably well with lift results for many aerofoils at incidences below stall. This is particularly so for those with shapes built around circular arc camber lines, provided that the thicknesses are arranged about that line so as to produce streamlined shapes on which the boundary layer remains attached until it enforces the Kutta-Zhukovskii condition at the sharp trailing edge.

The mathematical technique of conformal mapping, mentioned above, provides a method for carrying over the known velocity and pressure distributions of the cylinder-with-vortex flow field of Figure 14b to the transformed shape of interest, in this case the aerofoil. However, one further result carried over in this process is that which requires the drag to be precisely zero; although realistic results are obtained for lift, d’Alembert’s Drag Paradox cannot be escaped. However, once the boundary-layer concept had been accepted, it then became clear that for well-streamlined bodies the Paradox is almost correct: the drag is the Paradox’s zero value, plus a small correction contributed by the boundary layer’s viscous effects.

A rough idea of the conformal mapping procedure can be gained by imagining the flow field of Figure 14b sketched on a sheet of highly elastic material. By stretching the sheet appropriately, both the cylinder’s shape and its flow field can be imagined as being distorted to something like Kutta’s arc and Zhukovskii’s streamlined aerofoil.
Appendix 4

The Theorems of Stokes, von Helmholtz and Lord Kelvin

The title Stokes’ theorem may be rather a misnomer. It appears that William Thomson (1824-1907) (in 1892, Lord Kelvin) mentioned this theorem in a letter to George Gabriel Stokes (1819-1903) at Cambridge in 1850. The latter set the problem as part of the Smith’s Prize Examination there in 1854. The theorem reached a wider audience through later publications by, for example, Thomson \(^{(60)}\) in his paper on vortices. Although named Thomson’s theorem by some writers, Stokes’ theorem seems to be the title more widely adopted.

As stated in Section 2, the theorem relates a fluid’s vorticity within an enclosing circuit to the circulation around that circuit. In mathematical terms, the vorticity is defined as combinations of the velocity gradients at an arbitrary point in the fluid. Readers familiar with calculus will not be surprised to learn that, when all such velocity gradient combinations are summed by integration throughout the fluid within the circuit, the result is the line integral of the flow velocity component around that circuit, essentially as stated in Section 2. As an illustration of the method, this calculation is applied to boundary-layer flow at the close of Appendix 5.

In 1858 Hermann Ludwig Ferdinand von Helmholtz (1821-1894) published three theorems on vortices \(^{(61)}\). The first, relevant to our discussion of trailing vortices in Section 2, states that for such a case a vortex line or filament, once created, cannot end in the fluid and hence appears as a closed loop. In 1869 all three theorems were encompassed within the single theorem of Thomson which is usually referred to as Kelvin’s theorem \(^{(60)}\). This states that, for a fixed quantity of fluid possessing circulation, the circulation does not change as that fluid moves through the flow field. It should be added that, as originally derived, the Helmholtz and Kelvin theorems did not include the influence of viscosity.

In a crude sense you can see the basis of the above vortex theorems by looking again at the circular fluid element of Figure 6. In the absence of viscous shearing forces, the element’s weight and its surrounding pressure cannot induce the element to spin. If, on the other hand, it is already spinning then there is no mechanism to stop this.
Appendix 5

Viscous Flows: The Boundary-Layer Concept

As so often in the history of mechanics, one of the earliest steps to understand viscous action was taken by Newton\(^{(37)}\). He wished to explain why it is that a circular cylinder, set into rotation in a water bath, creates a vortex about itself. He realised that, if the fluid is to start moving, it must stick to the cylinder’s rotating surface; this is the earliest application of the no-slip condition mentioned in Section 2. The rest of the fluid, he argued, is then set in motion by what he called a “want of lubricity” between concentric fluid layers. The resulting rubbing action between any one layer and its neighbours, he proposed, was proportional to the rate at which the flow velocity varies radially through the fluid. Unfortunately, this hypothesis is incorrect in this case so that his solution to the problem is in error.

Newton’s difficulty here was a lack of fundamental concepts which took rather more than a further century to emerge. Major steps here were taken by Baron Augustin-Louis Cauchy (1789-1857) between 1822 and 1843\(^{(36)}\). Firstly, he generalised Euler’s concept of pressure, the stress perpendicular to a surface, so as to include tangential shear stresses. Secondly, he showed that distortions to a moving fluid element’s shape can be expressed as the rates of change of the flow velocity components with distances through the flow, in other words to the velocity gradients within the flow. Certain combinations of velocity gradients were seen to give the rates of strain, other combinations the vorticity. In Britain, it was William Thomson’s initial interest in electricity and magnetism which drew him to the field theory’s extension to fluid flow and its above developments. This involvement led to his basic exercise in calculus applied to vorticity which resulted in the Stokes theorem of Appendix 4.

Other French mathematicians such as Poisson and Barré de Saint-Venant became involved in these activities. Meanwhile, Claude-Louis-Marie-Henri Navier (1785-1836), using a molecular model of a fluid, obtained relations\(^{(62)}\) for shear stresses which he then added to Euler’s field-theory equations outlined in Appendix 2. In 1845 Stokes\(^{(63)}\) arrived at essentially the same equations as Navier for fluids possessing what he termed “internal friction”. Here, however, as Stokes\(^{(63)}\) pointed out, whereas stresses in a solid substance are proportional to strains, in a moving fluid its shearing stresses are proportional to \textit{rates} of strain. At this stage, the ‘constant of proportionality’ in the latter relations was taken to be a property of the fluid, like density, but as yet remained unidentified. With the emergence of the kinetic theory of gases in the 1860s, this ‘constant of proportionality’ was finally seen to be the viscosity coefficient. Stokes\(^{(63)}\) found solutions to what are now called the Navier-Stokes equations for only a few cases involving simple geometrical configurations. One was flow in a long pipe, another that in a rotating annulus and here he corrected Newton’s error in his rotating cylinder problem, pointing out that the rate of strain in that case was more complicated than Newton\(^{(37)}\) had, in effect, assumed.

Because of the Navier-Stokes equations’ mathematical complexity, a limited number of analytical solutions to them have been found (see Reference 64). However, a major breakthrough came with Prandtl’s announcement of the boundary-layer concept at the Third
International Mathematical Congress at Heidelberg in 1904 (65). His basic point was that, for fluids of very low viscosity, the viscous effects described by the Navier-Stokes equations are often confined to a very thin layer at a body’s surface, the rest of the flow behaving as if inviscid. He showed that, for such thin boundary layers, the Navier-Stokes equations take on a simplified and far more tractable form. One important result here is that the pressure does not change through the layer. A further result is that the predominant rate of strain is equal to the rate at which the velocity component parallel to the surface varies in the direction perpendicular to the surface, in other words to the velocity gradient as shown in Figure 18.

Thus the shearing stress at any point in the boundary layer is the multiple of the viscosity coefficient and this velocity gradient. Whereas that coefficient is very small indeed for air, for example, the velocity gradients in the boundary layer are extremely larger, resulting in sizeable values for the shearing stresses in the boundary layer. Exterior to the boundary layer, in contrast, the velocity gradients are relatively small so that, when multiplied by that same exceedingly small viscosity coefficient, the shearing stresses are negligible and that part of the flow can be calculated using the inviscid flow methods outlined in Appendices 2 and 3. As to the boundary-layer fluid’s vorticity, this too is equal to that same large velocity gradient at any point in the boundary layer. This feature will be returned to at the close of the appendix as an illustration of the derivation of Stokes’ theorem.

Taking the simplest case of the boundary-layer flow over a flat plate set at zero incidence, Prandtl (65) obtained a theoretical result for its skin friction drag which, when put in terms of the drag coefficient $C_D$, is

$$C_D = 2.2 \left( \rho V l / \mu \right)^{-1/2},$$

where $C_D = D / \left( \frac{1}{2} \rho V^2 S \right)$. (9)

Here $D$ is the skin friction drag force contributed by both faces of the plate of planform area $S$ and stream-wise length $l$. The flow velocity is $V$, the fluid has density $\rho$ and viscosity coefficient $\mu$. The numerical value of the above coefficient, 2.2, he found by rough calculation. As to improving on that calculation, he recommended the use of Kutta’s method for the numerical solution of differential equations mentioned in Appendix 3.

The flat plate case has no pressure change along the surface, but Prandtl went on to describe the effect of pressure variations along surfaces. In particular, he described the case in which the pressure in the inviscid flow exterior to the boundary layer rises along the surface. In this case, according to the Bernoulli equation (4), the flow velocity there must be decreasing. This deceleration on the exterior flow’s part affects the boundary layer, and all the more so since it is already retarded by its viscous forces. A consequence can be that the boundary-
layer fluid close to the surface simply stops moving altogether, the layer then separates from the surface and back flow occurs aft of the separation point. Prandtl’s drawing of this separation process is shown in Figure 19. Aft of separation, a wide wake possessing lower than ambient pressure is formed. This creates higher drag – the body is effectively sucked backwards – and, in the case of the aerofoil, a loss of lift occurs. From Prandtl’s work \(^{(65)}\) it at last became clear why streamlined shapes should be adopted if high drag is to be avoided; the use of a gently tapered tail, as seen in Figure 3, allows the pressure rise along the aft part of a body to be as gradual as possible so as to avoid excessive flow deceleration which results in separation.

![Figure 19](image1.png)

**Figure 19** Boundary-layer separation (Prandtl \(^{(65)}\), 1904).

At the Heidelberg Congress in 1904, Prandtl’s description \(^{(65)}\) of his work included observations of boundary-layer behaviour using a small water channel in which the flow was created by a hand-driven paddle wheel (Figure 20). One case observed was that in which the layer separates at the sharp edge of a plate, another the layer’s separation from the surface of a circular cylinder. In the latter case, the body’s rearward contour is too blunt and the pressure rises too rapidly. However, in this case Prandtl showed that separation could be suppressed by suction applied at the cylinder’s surface.

![Figure 20](image2.png)

**Figure 20** Water channel (Prandtl \(^{(65)}\), 1904).
Prandtl’s discovery of the boundary layer took place at the Technische Hochschule, Hannover. In 1905 he moved to Göttingen University, there to head an institute which later became dedicated to aerodynamic research. One of his earliest doctoral students was Paul Richard Heinrich Blasius (1883-1970) who published his analytical solutions to a number of boundary-layer problems in 1908. The first case considered is Prandtl’s earlier problem of the boundary-layer development over a flat plate. Blasius’ more accurate solution not only confirmed the form of Prandtl’s expression for skin friction drag, equation (9), but also produced a much improved value, 2.654, for the coefficient involved in that equation. Subsequent analyses of even higher accuracy succeeded in changing only slightly the Blasius value to 2.656.

One of the other boundary-layer problems analysed by Blasius is that around a circular cylinder. For this he assumed the exterior flow’s pressure distribution to be that given by inviscid flow theory, specifically the flow field shown in Figure 14a. His calculation had the boundary layer in this case separating at an angle of about 110° from the nose, a result at variance with experiment. In such circumstances, however, the large wake formed aft of separation so grossly distorts the inviscid flow field that the assumed exterior flow’s pressure distribution is invalid. In 1911 Karl Hiemenz revisited this calculation using experimentally measured values for the cylinder’s pressure distribution. Having calculated the boundary layer’s development in these circumstances, he found that his theoretical prediction for separation gave this at 82° from the nose, whereas experiment had this point at 81°.

In 1914 Prandtl reported his experimental investigation of the flow around a sphere. Usually the boundary layer over the forward part of a body is smooth and in what is called laminar motion. Prandtl found that in the sphere’s case the laminar boundary layer separated a little before the sphere’s equator, a situation similar to the cylinder case of Hiemenz. Prandtl then attached a fine wire to the sphere’s forward surface, the consequence being that the laminar boundary layer was tripped by the wire into highly irregular turbulent motion. He found that the turbulent boundary layer then separated further aft and on the rearward face of the sphere. This created a narrower low-pressure wake which resulted in a significantly reduced drag on the sphere.

From this and subsequent experiments it emerged that laminar boundary layers, when subjected to slight disturbances such as surface irregularities or dirt in the flow, all of which are difficult to avoid, have a tendency to trip into turbulence. This results in increases in both boundary-layer thickness and in skin friction in that locality. As Prandtl observed with his sphere, however, usually turbulence is avoided when the exterior flow is accelerating over the forward part of a body, the pressure dropping along the surface. But in the constant pressure case of the flat plate, turbulence occurs at some distance aft of the leading edge. And in regions of rising pressure, which will always be present over the rearward parts of bodies, the boundary layer is particularly prone to turbulence.

The situation at the rear of streamlined bodies is additionally complicated by the possibility of boundary-layer separation. As noted earlier for the laminar boundary layer, if the aft pressure rise is too rapid then separation may well ensue. Alternatively the laminar boundary layer may trip into turbulent motion. However, although a laminar boundary layer creates
smaller skin friction but has a tendency to trip into turbulence in regions of adverse pressure increases, a turbulent boundary layer is more resilient in resisting separation. In any particular situation, how much of the boundary layer is laminar, how much is turbulent, and whether or not separation is occurring depends on the size and speed of the body in question. This problem of what was initially called ‘scale effect’ in Britain is discussed in Appendix 7.

Earlier it was remarked that in the boundary layer the vorticity is simply the velocity gradient shown in Figure 18. To illustrate its use in the derivation of Stokes’ theorem, we take x as the ordinate along an aerofoil’s surface, u being the velocity component in that direction, whilst y is the ordinate perpendicular to the surface. In calculus notation the velocity gradient at any point within the boundary layer is then $\frac{\partial u}{\partial y}$, and this is also the value of the vorticity which exists at an elementary area $dydx$. Following the procedure outlined in Appendix 4 to obtain Stokes’ theorem, we first calculate the sum of the boundary layer’s vorticity in an arbitrary y-wise strip from the surface at $y = 0$ to the boundary layer’s exterior edge. This summation is $\int \frac{\partial u}{\partial y} dy$ and yields the value of $u$ at the boundary layer’s exterior edge, $u_e$, minus its value at the surface $y = 0$. The latter, by the no-slip condition, is zero. Thus the total vorticity within this y-wise strip of width $dx$ is $u_e dx$. To calculate the total vorticity within the whole of the boundary layer, we must then sum all such values of $u_e dx$ throughout the complete range of x. This is $\int u_e dx$, the line integral of Stokes’ theorem mentioned in Appendix 4. It is to be calculated around a complete circuit enclosing all of the boundary layer’s vorticity.

To return to the fundamental point made in Section 2, random molecular motion creates both the surface no-slip condition and the viscous action within the boundary layer. These, in combination, create the boundary layer’s velocity gradients. The latter, on the one hand, are the source of the vorticity which generates the circulation and hence the lift. On the other hand, the multiple of the velocity gradient’s value at the surface and the viscosity coefficient determines the value of the skin friction drag.
Appendix 6

Non-Dimensional Coefficients

In his study of the interaction between a flowing fluid and a solid body, Newton\(^\text{(37)}\) showed that a continuous momentum change will occur in the fluid either by it being deflected around the body or by direct collision with the body. Thus for a body of characteristic area S exposed to a flow of velocity V in a fluid of density \(\rho\), the resistance \(R\) experienced by the body due to this interaction will take the form:

\[
R \text{ is proportional to } \rho V^2 S.
\]

Newton’s statement\(^\text{(37)}\) is not only correct but, more importantly for present purposes, it is also dimensionally correct. If one says that a force, in this case \(R\), can depend only on \(\rho\), V and S, then the only combination of these quantities which has the dimension of force is \(\rho V^2 S\). To generalise, a quantity having a specific dimensional unit on the left hand side of an equation must be balanced by a combination of quantities on the right hand side which also has that same specific dimensional unit. The specific dimensional unit here is that of force and, in the International System of Units (SI) adopted in Britain, the unit of force is the newton (= kg.m/s\(^2\)). The combination \(\rho V^2 S\) has dimensional units \((\text{kg/m}^3)(\text{m}^2/\text{s}^2)(\text{m}^2)\)\(^\text{[= kg.m/s}^2\text{]}\), which is the same dimensional unit as force. However, to make a more fundamental point, the laws of physics are entirely independent of how we choose to measure things, that choice being agreed between mere mortals. Thus any theoretical result obtained on the basis of those laws cannot depend on any specific choice of man-made units. Moreover, such results are most economically displayed in non-dimensional form. By the same token, it is sensible to display experimental results in forms which do not depend on the system of units used in making the measurements; that is we should present experimental results in non-dimensional forms.

Take Bernoulli’s equation (4) as an illustration: the quantity \(\frac{1}{2}\rho v^2\) must have the same dimensional unit as the pressure, \(p\). Thus the vertical ordinate of Figure 4, labelled \((p - p_0)/\frac{1}{2}\rho U_0^2\) (\(U_0\) being the free stream velocity otherwise denoted by \(V\) here), has no dimensional unit but purely the numerical values shown. The same non-dimensionality applies to Figure 4’s horizontal ordinate, \(x/c\), which is the distance, \(x\), from the leading edge divided by the aerofoil’s chord, \(c\). In order to provide Figure 4, doubtless Tanner\(^\text{(3)}\) took his measurements using the old British system of units (lb, ft, sec) but he displayed his data non-dimensionally.

In 1910 Lord Rayleigh\(^\text{(69)}\) took the argument for the resistance \(R\) a stage further by adding the possibility that it might also depend upon the fluid’s viscosity coefficient, \(\mu\), and some characteristic length scale for the body, \(l\). Using the dimensional argument outlined above, he arrived at the statement:

\[
R \text{ is still proportional to } \rho V^2 S,
\]

but is now also proportional to something which depends on \((\rho V l / \mu)\).
This last combination of quantities has no dimensions and was later named the Reynolds number, Re, so that

$$\text{Re} = \frac{\rho V l}{\mu}.$$  \hfill (10)

This attribution celebrates the experimental work on pipe flow reported (70) in 1883 by Osborne Reynolds (1842-1912) of Owens College, Manchester (later the Victoria University of Manchester), in which he found that at low values of this non-dimensional quantity the pipe flow was laminar whereas at higher values the flow became turbulent. More generally, the conditions at which laminar boundary layers trip into turbulence, as described in Appendix 5, are also dependent on the value of Re.

To return to the force R, we are more interested here in its two components, lift L, perpendicular to the free stream direction, and drag D, in the free stream’s direction. On the basis of Lord Rayleigh’s statement (69), we can express these in terms of two non-dimensional coefficients,

$$C_L = \frac{L}{\frac{1}{2} \rho V^2 S} \quad \text{and} \quad C_D = \frac{D}{\frac{1}{2} \rho V^2 S},$$  \hfill (11)

so that Lord Rayleigh’s statement becomes

$$C_L \text{ and } C_D \text{ depend on Re.}$$

The factor of $\frac{1}{2}$ is included in the definitions of $C_L$ and $C_D$ because $\frac{1}{2} \rho V^2$ can conveniently be measured by a Pitot-static tube either in wind tunnel tests or on a full-scale aeroplane (see Appendix 2). Care must be taken, however, when reading older reports on aerodynamic tests since, prior to the early 1930s, the convention was to exclude the factor of $\frac{1}{2}$. Moreover, many early ACA reports do not state the value of Re at which experiments were conducted, merely giving the value of $Vl$ since the density and viscosity values would not vary significantly between the various wind tunnels used for testing in those days.

Lord Rayleigh’s general result for the resistance R, quoted above, implies that both $C_L$ and $C_D$ should depend on Re. If we re-cast equation (9) for the flat plate laminar boundary-layer case of Blasius (66) as

$$C_D = 2.656 \text{Re}^{-1/2},$$  \hfill (12)

then this clearly shows this dependence. Indeed, all laminar boundary-layer skin friction results follow this dependence on Re$^{-1/2}$. A laminar boundary layer’s thickness also varies as Re$^{-1/2}$; thus the higher the Reynolds number, the thinner is the boundary layer. This points to the fact that the thin boundary layer is fundamentally a high Reynolds number phenomenon. Turbulent boundary layer results, however, have rather different forms giving thicker boundary layers and higher drag which varies more gradually with Re.

As to the $C_L$ results given in equations (7) and (8) of Appendix 3, in contrast, no dependence on Re is evident. Although viscous action in the aerofoils’ boundary layers is the basic cause of the circulation and hence the lift, once the boundary layer has controlled the exterior inviscid flow field by enforcing the Kutta-Zhukovskii condition at the trailing edge, that lift-
producing flow field exhibits scarcely any dependence on viscosity and hence on Re. This conclusion also applies to the induced drag coefficient since it is determined entirely by the wing’s lift. This argument breaks down, however, when the boundary layer can no longer enforce the Kutta-Zhukovskii condition, such as in the case of aerofoil stall. In that circumstance $C_L$ does exhibit a dependence on Re, as we shall see in the next appendix.

In Appendix 3 the statement was made that, for well-streamlined aerofoils on which the boundary layer remains attached, the drag is the d’Alembert Paradox result of zero plus a small correction from the boundary layer. To obtain a rough idea of how small this correction can be, firstly it is necessary to understand that, because of the extremely low value of the air’s viscosity coefficient, aeroplane flight takes place at very high values of Re. Even for the small wind tunnel model of the Fokker wing described in Appendix 7, the test value of Re was a little under $10^6$ and for full-scale aeroplanes this can be as high as $10^8$. In the skin friction drag result for laminar boundary layers, equation (12), the crucial factor is $Re^{-1/2}$ and for a value of $10^8$ for Re this factor is 0.0001. However, as remarked in Section 2, for aerofoils an additional drag occurs due the slight pressure imbalance created by the boundary layer’s small thickness at the trailing edge – the ‘shouldering aside’ effect on the exterior flow mentioned there. This adds a comparable amount to the drag but, even so, the boundary layer’s net contribution to drag is still small. Regrettably, however, such drag results drawn from laminar boundary layer behaviour are unrealistic at Re values as high as $10^8$ since, for full-scale flight at such values, a sizeable rearward portion of the boundary layer would be turbulent. For this Re range, in broad terms the drag factor is raised by a multiple of ten, yet even so the resulting total drag due to the boundary layer is still fairly small.

If we exclude induced drag but include all of the boundary layer’s pressure and skin friction drags contributed by the complete airframe of the modern aeroplane, the total drag can be raised by a further multiple between five and ten. Using this crude argument, a modern, well-streamlined aeroplane might be expected to yield a drag coefficient, $C_D$, in the region of 0.01. Here the characteristic area, S, used in the definition of $C_D$ is the wing planform area. Loftin states a value of 0.0119 for the Martin B-57B, the American version of the English Electric Canberra. This aerodynamically clean aeroplane, being jet-powered, avoided the additional drag penalties of piston-engined aeroplanes caused mainly by their engine-cooling systems. Probably the lowest $C_D$ value achieved in the latter case was that for the North American P-51D, for which Loftin gives the value 0.0163. These values can be contrasted with the drag coefficient of a very unstreamlined object such as a circular cylinder for which the drag coefficient is about the order of unity. However, the quest in aerodynamics is often not so much to minimise the drag whilst achieving adequate lift but rather to maximise the lift to drag ratio. Two examples illustrate this ratio’s importance. For gliding flight in still air, it turns out that the higher the lift to drag ratio the shallower is the gliding angle. For powered flight, the higher the lift to drag ratio the greater is the aeroplane’s range. Modern high-performance gliders can achieve lift to drag ratios of 40 or more through a combination of exceptionally clean aerodynamic shape and wings of large area which are also of very high aspect ratio. The latter reduces the induced drag coefficient since this turns out to be proportional to the reciprocal of aspect ratio. For the modern airliner with its bulkier shape and wing aspect ratio, though high, limited by structural constraints, the lift to drag ratio at
cruise conditions can approach a value of 20.
Appendix 7

‘Scale Effect’: The Confusing Case of the Fokker Wing

During the First World War, two German fighter aeroplanes were found to be particularly formidable adversaries. These were the Fokker Dr 1 triplane and the D VII biplane. Both aeroplanes had wings in which the structural stiffness was built within the wings themselves, thus avoiding the use of drag-producing interplane bracing wires. To accommodate their substantial stiffening structures, the wings possessed a rather thick aerofoil section. It was suspected that this shape produced the added benefit that the aeroplanes could be taken to relatively high angles of incidence before stalling. To investigate this possibility, a model of a thick-section Fokker wing was tested at the NPL by Bryant and Batson (72), the results being released in 1919. The wing was small, having a chord of 3 in and a span of 18 in, the aspect ratio thus being 6.

The tests were conducted at three tunnel speeds, 22.2 ft/sec, 40 ft/sec and 60 ft/sec, the corresponding Reynolds numbers being $0.35 \times 10^5$, $0.63 \times 10^5$ and $0.94 \times 10^5$. The results for the variations of the measured lift coefficients with incidence angle are shown in Figure 21, together with the Fokker wing’s aerofoil section shape. Here it should be noted that the old definition of lift coefficient is used, which is $C_L/2$ in modern notation. At small incidences all three graphs coincide and follow the sin $\alpha$ behaviour of equation (8). Thus, as suggested in Appendix 6, these lower incidence results show no evidence of a dependence on Re. However, at the lowest test speed stall occurs at a relatively small incidence whereas at the two higher speeds stall is delayed until a lift coefficient is reached which is almost double that of the lowest speed case. At that time, Bryant and Batson (72) could offer no explanation for this remarkable behaviour, only recommending further tests at even higher $V_l$ conditions, that is at higher speeds and with a larger model, in other words at higher Reynolds numbers.

As pointed out in Section 3, what was lacking at that time in Britain’s debate on the ‘scale effect’ $V_l$ was an understanding of boundary-layer behaviour. Had that been available, it would have been understood that, at the lowest test speed, laminar separation was taking place at quite small incidences. At the higher speeds and therefore higher Reynolds numbers, in contrast, the boundary layer had tripped into turbulence at some point on the aerofoil’s surface. Better able to withstand the strong rearwardly rising pressures at these higher incidences, this turbulent boundary layer had ensured that the wing could be taken to a significantly higher incidence before separation occurred and stalling behaviour appeared. This clarification of the aerofoil’s behaviour can be found in Reference 4 (pg. 465). As a general point, however, the test results show that the value of the Reynolds number can have a dramatic influence on lift results so far as stalling behaviour is concerned.
Figure 21   Lift results for Fokker aerofoil (Bryant and Batson (72), 1919).